

# Modern and Intuitive Way of Teaching Space Vector Calculus and PWM in an Undergraduate Course

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## Abstract

In the field of power electronics and electrical machines space vector theory is of paramount importance for a detailed understanding of three-phase systems. However, students frequently do have problems with getting a clear picture of all advantages of the space vector concept and, therefore, often do consider space vector calculus to be of limited practical applicability. A part of the new web-based educational software iPES (Interactive Power Electronics Seminar, available in English, German and Japanese at [www.ipes.ethz.ch](http://www.ipes.ethz.ch)) focusing on space vector calculus could provide a solution to this problem.

Four representative Java applets dealing with the main aspects of space vector calculus are described in detail in this paper. The applets do allow the student to explore the basic rules of space vector transformation in an interactive and animated way. Furthermore, the student does get experience with the application of space vector calculus for analysing a three-phase PWM converter, and finally has to manage space-vector-based current control of a three-phase PWM converter system. All applets are easy to use and do provide a smooth introduction into the topic also for beginners.

## 1 Introduction

Using new media and IT-technologies in the classroom can not only make studying more attractive to the student, but might make also teaching much easier. Especially in engineering classes complex technical problems have to be presented in a way which is easy to follow and understand. If, additionally, the system to be described shows dynamic behavior and/or a lot of different possible states do occur teaching becomes very hard for the instructor in case only words and static pictures are at hand. Here, interactive animation (as shown in the following) might help a lot.

Another major advantage of E-Learning is the independence of working place and time. The student

has the opportunity to learn in his favorite environment at the time of his choice. This might not be seen as a great advantage for the university student, but it is especially important for the working engineer who wants to update his knowledge by taking E-Learning lessons in the evening after work and on the weekends at home.

In this paper an example of E-Learning for an effective introduction into the space vector theory for three-phase PWM converter systems and electrical machines is given. The E-Learning software focusing on space vector calculus is part of iPES (Interactive Power Electronics Seminar) that is available on the Internet at [www.ipes.ethz.ch](http://www.ipes.ethz.ch). iPES is a collection of Java applets for interactive animation and simulation of power electronic systems that is used in addition to more conventional teaching in the basic power electronics course at the ETH Zurich.

Due to the employed technology the usage of iPES in education is extremely flexible. Generally, the tool can be used for distance learning via the Internet, in more traditional lessons in the classroom (with or without the use of other multi-media tools), or for self-studying at home.

## 2 Teaching space vector calculus in power electronics

The space vector calculus is very convenient for describing and understanding the behavior of three-phase PWM rectifier systems and electrical drives. Instead of analyzing the time-behavior of three-phase quantities which are mutually dependent (for three-wire systems), the phase quantities are transformed into one time-dependent vector described in the complex plane ("space vector") according to

$$\underline{u}_N(t) = 2/3 [ u_{N,R}(t) + \underline{a} u_{N,S}(t) + \underline{a}^2 u_{N,T}(t) ]$$

where

$$\underline{a} = \exp(j 2/3 \pi)$$

is a complex constant,  $u_{N,R}(t)$ ,  $u_{N,S}(t)$  and  $u_{N,T}(t)$  are the three phase quantities and  $\underline{u}_N(t)$  is the resulting space vector.

Teaching space vector calculus is frequently done in a quite abstract and theoretical style. Students, starting studying three-phase systems or drives, are told at first mainly just mathematical aspects of the space vector theory. This would be accepted by the students in a course on mathematics but is often disliked in an engineering course where the students expect to see concepts of immediate practical applicability.

The basic problem is that space vector calculus is somewhat abstract and therefore needs to be studied in detail. In order to make studying and also teaching of space vector theory more attractive a part of iPES has been dedicated to space vector calculus.

There, the main idea is that the student can change the time behavior of the phase quantities by simply dragging curves with the mouse and does immediately see the effect on the space vector or vice versa (section 3.1 and 3.2).

Another part is the application of space vector theory on a converter system where the student has to perform the current controller's task (section 3.4). The student has to select converter switching states in order to control the inverter phase currents. This can be done intuitively by watching all three phase currents but the student learns within a minute that it is much more effective to make the switching decisions based on the voltage space vectors. The appearance is similar to a computer game what does get the student emotionally involved and accordingly does show a high learning effect.

It is important to note that the applets are intended to show the student how space vector theory works and how it can be used for building power electronics systems. It is still necessary to learn the mathematics by using a textbook, and paper and pencil in the traditional way, but it does show the student from the very beginning how the abstract theory can be used for solving practical problems. So, adding the applets to traditional teaching might motivate students and allow to speed up the learning process.

Some representative Java-applets for interactive animated teaching of space vector calculus are presented in the following. The applets are inserted seamlessly into the explaining HTML text and do offer a very simple and self-explaining graphical user interface. The whole E-Learning software tool iPES does contain of a set of more than 25 Java-applets like the following examples and does form a full one-semester introductory course on power electronics.

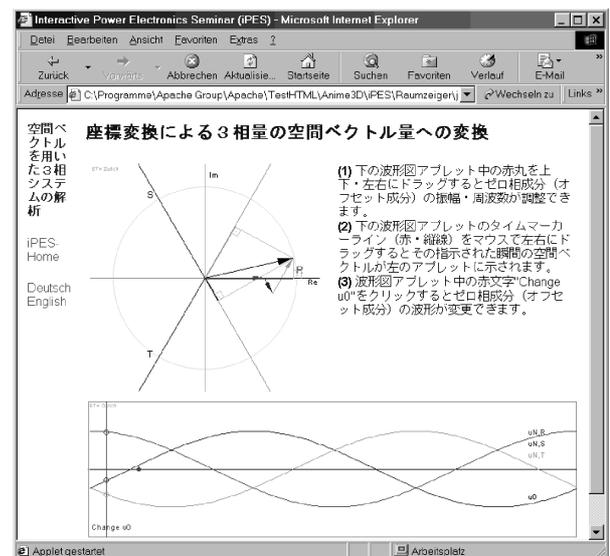
The graphical user interface of the applets is very intuitive. The general rule (with some exceptions) is: One can click or drag all red things with the mouse. The HTML text explains only what can be done with the mouse, not what is being shown. Explaining the content is still the job of the teacher or of an additional textbook.

### 3 Interactive animated applets for teaching and studying space vector theory

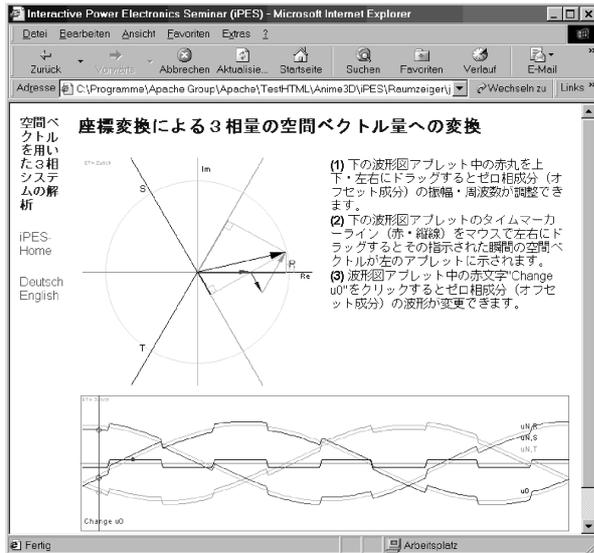
#### 3.1 Transformation of phase quantities into a space vector

At first the basics of the transformation rule have to be understood. All the important details are visualized and the student can interactively influence parameters.

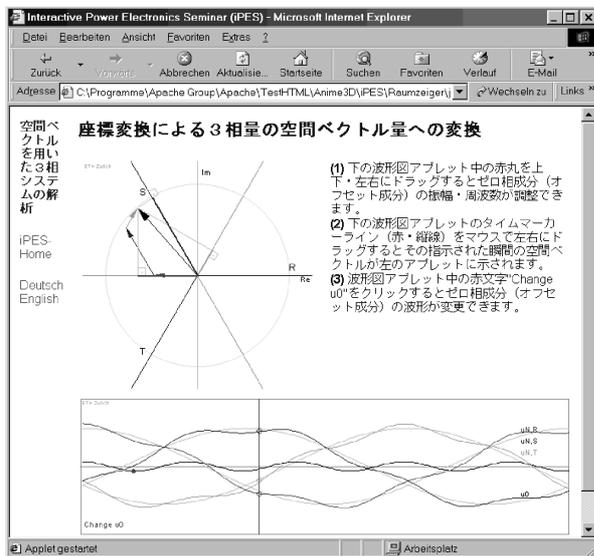
The applet in Fig.1(a) shows the transformation of phase quantities into a space vector. The time-behavior applet shows a pure sinusoidal three-phase system with phase quantities R, S, and T. A time-marker (vertical red line) can be dragged with the mouse for defining a certain point in time. Transforming the phase quantities at the defined time into a space vector results in the picture shown in the upper applet. Here, the projections of the space vector onto the axes R, S, and T are drawn in the same colors as the phase quantities in the time behavior applet. Linear dragging of the time-marker with the mouse results in a rotation of the space vector in the upper applet.



(a)



(b)



(c)

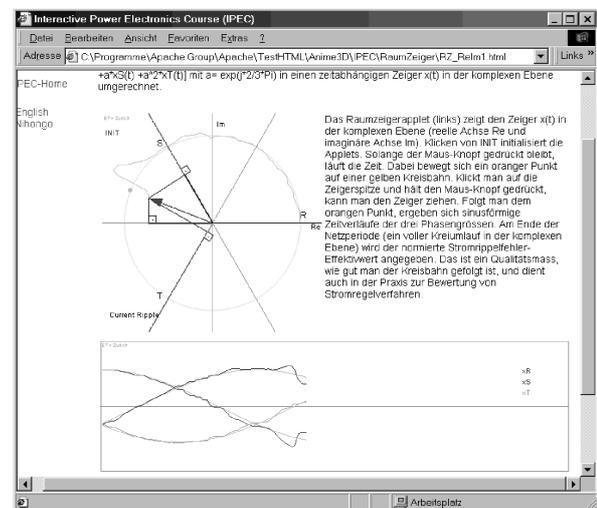
**Fig.1:** (a): Transformation of three phase quantities defined by a time marker (vertical red line) into a space vector; (b): adding a periodic rectangular offset to all three phase quantities does not influence the resulting space vector; (c): changing the offset signal to a sinusoidal time behavior.

By dragging the red dot on the time-axis in the vertical direction an offset value  $u_0$  can be added to all three phase quantities. While in **Fig.1(a)** the amplitude of  $u_0$  is close to zero, in **Fig.1(b)** the offset in form of a periodic rectangular signal can be clearly seen. Due to the space vector calculus such an offset being equal to the three phase quantities does not influence the space vector. This can be shown easily with the applets. Changing the amplitude of  $u_0$  does not effect the resulting space vector but only its components according to the phase quantities (these are also shown in the space vector applet).

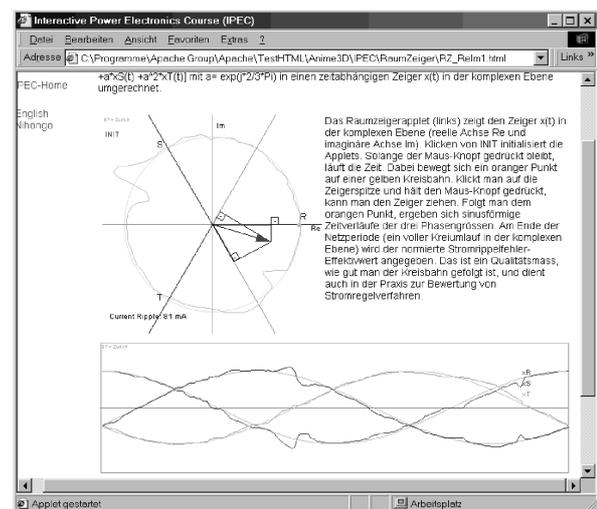
By clicking the red text “Change  $u_0$ ” in the time behavior applet the form of the periodic offset signal can be changed from rectangular into sinusoidal. This is shown in **Fig.1(c)**.

### 3.2 Calculation of the phase quantities related to a space vector (back-transformation)

After having been provided with a clear picture of the basics of the transformation rule the student does proceeds with the problem how a sinusoidal three-phase system results in a rotating space vector, and how the ripple in the time behavior results in deviation of the space vector trajectory from a circular shape.



(a)



(b)

**Fig.2:** Trajectory of the space vector within one mains period (a). After one mains period (b) the rms-value of the current ripple of one phase quantity (deviation from pure sinusoid) is calculated. The languages describing the applets can be selected as German, English and Japanese.

The space vector applet (top) of **Fig.2(a)** shows the space vector in the complex plane where the vector can be dragged with the mouse and the vector's projections are transformed into the three time-dependent components which are depicted in the time-behavior applet (bottom). The three projections and/or phase quantities in the space vector applet and the according curves in the time-behavior applet are shown in the same color.

The interactive animation is started with a mouse-click. Then, the space vector has to be dragged with the mouse to follow an orange dot that is moving along a circle (according to the theory this results in three pure sine-waves). The curves that result from dragging the space-vector with the mouse are also shown in the time-behavior applet. After one main period the interactive animation stops and the deviation of the currents from the ideal sine-waves is calculated (see **Fig.2(b)**). This is a measure for the quality of dragging the space vector with the mouse (as it is also used in praxis to evaluate the quality of a current controller, for example).

The student has to guide the space vector with the mouse and his overall error (deviation from pure sine) is reported. Doing this the coupling between space vector in the complex plain and the time-behavior becomes immediately obvious to the student. Also the transformation rules can be understood easily.

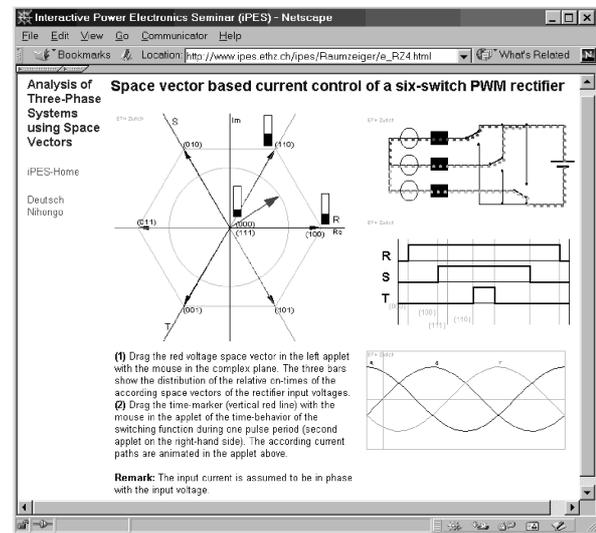
### 3.3 Space vector based current control of a six-switch PWM rectifier

Now the student has to apply space vector calculus to an inverter system. The mutual relations of voltage space vectors, switching patterns and input currents are visualized.

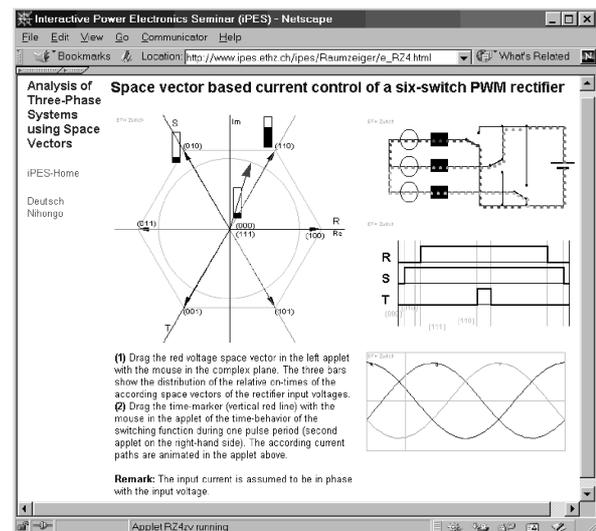
**Fig.3** shows screenshots from the applet that is dealing with space vector based current control. The rectifier system is controlled with a PFC current control scheme.

The student can drag the mains voltage space vector with the mouse in the complex plane (left-hand applet). Only the rectifier voltage space vectors forming the triangle around the mains voltage space vector are used to guide the current in order to minimize switching losses. The left-hand applet shows the normalized turn-on times of the rectifier voltage space vectors in form of three black bars in rectangular boxes. In **Fig.3(a)** the actual mains voltage space vector is close to the middle of the triangle, so all three vectors contribute approximately equally. In **Fig.3(b)** a different position of the mains voltage space vector is shown. Because it is close to the voltage space vector  $\underline{u}_U(110)$ , the according bar shows a much higher contribution than the other two

vectors  $\underline{u}_U(111)$  or  $\underline{u}_U(000)+\underline{u}_U(100)$ . This means that the on-time of the switching state (110) is significantly longer than on-time for (100) or (000)+(111).



(a)



(b)

**Fig.3:** Space vector based current control of a six-switch converter. (a) Voltage space vector of the mains voltage for  $0 < \varphi_U < \pi/3$ , actual switching state is (110). (b) Voltage space vector of the mains voltage for  $\pi/3 < \varphi_U < 2\pi/3$ , actual switching state is (010).

While in the left-hand applet the three bars give the distribution of the switching states of the system, the top right-hand applet shows the power circuit of the system. The current flow is animated by colored moving dots. The symbols of the switches are drawn in on- or off-state. The applet below the power circuit applet shows the time behavior of the switching signals of the three phases during one switching period  $T_p=1/f_p$ . There is a vertical red line that can be dragged with the mouse (time marker) that lets the

user define an actual time. The currents paths in the power circuit applet above are animated according to the position of this time marker.

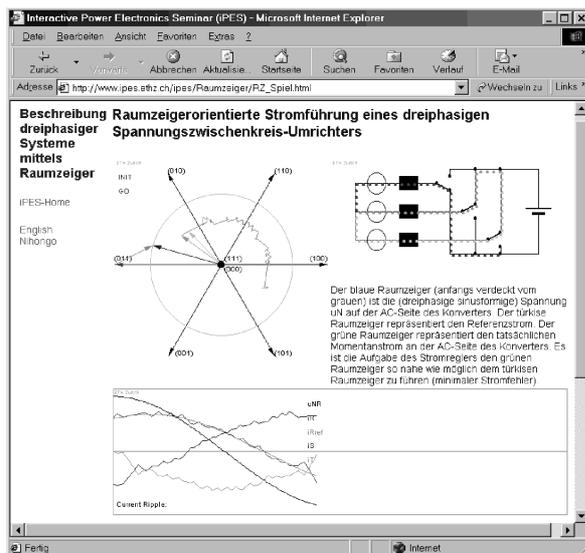
This approach is very useful in explaining the basic function of a power circuit and is a main technique employed in iPES. The user is free to choose a certain time within a given time behavior and the currents in the picture of the power circuit are animated accordingly.

The bottom right-hand applet shows the time-behavior of the mains voltages with a vertical marker. This time marker shows the actual position within one mains period due to the mains voltage vector in the left-hand applet.

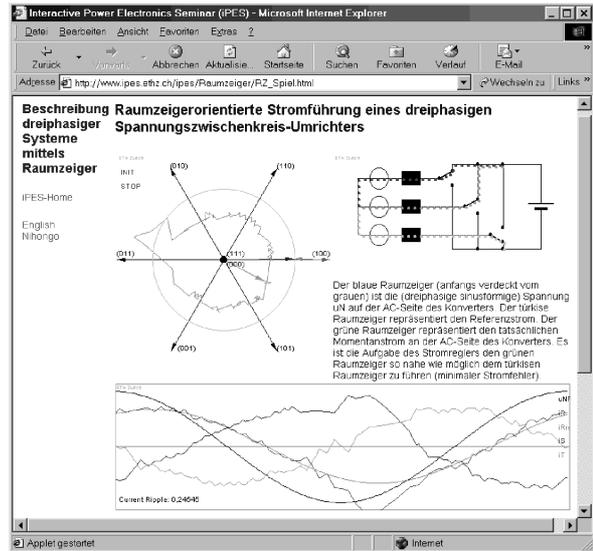
Technically speaking, there are four different applets on the web page shown in **Fig.3** that are all connected to each other. This means, that changing the mains voltage space vector or the time marker by dragging with the mouse will result in immediate updates of all data in all these connected applets.

### 3.4 Space vector based current control in a three-phase voltage DC link inverter

Finally, the student has to perform the current controller's task and has to control the three PWM rectifier input phase currents. Running a mains period does take less than two minutes and the student does get a clear understanding that space vectors are very useful and convenient for controlling three-phase power electronic systems.



(a)



(b)

**Fig.4:** Space vector based current control in a three-phase voltage DC link inverter. (a) During one mains period. (b) One mains period finished, rms-value of the inverter input current ripple is given (time-behavior diagram, bottom left).

The according applet is shown in **Fig.4**. During one full mains period, while time is running, the student continuously has to select one of the eight inverter voltage space vectors in order to guide the inverter AC side current along the three-phase sinusoidal reference currents with a minimum current error in all three phases. The left-hand applet shows the space vectors and the trajectory of the actual current space vector. There is also the reference current space vector moving on a circle that defines the ideal trajectory. The right-hand applet shows the power circuit with the animated current paths according to the actual situation. The time-behavior applet at the bottom shows the three inverter AC side currents, the sinusoidal reference currents, and the main voltage of one phase.

Running one mains period takes less than two minutes. During this time the student learns that switching states are associated with certain inverter voltage space vectors. Next, the student learns that in order to minimize current ripple the selection of the next voltage space vector has to be quick which means high switching frequency. Next, the student learns that choosing only the voltage space vectors in the vicinity of the mains voltage space vector keeps the current ripple low. After finishing one mains period the student knows how such a current controller works. It is the teacher or the textbook's job to explain all the details but this applet helps the student within a very short time to get a good feeling how such control schemes work.

After one mains period is finished, the normalized rms-value of the current ripple of one phase is given

(Fig.4(b)). This is a quality criterion of the controller's design or, as given here, of the student's understanding of a current controller's task.

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## 4 Summary

This paper does provide a detailed introduction into a part of iPES (Interactive Power Electronics Seminar - a new non-commercial educational software available at [www.ipes.ethz.ch](http://www.ipes.ethz.ch)) which covers space vector calculus.

The educational software iPES (Interactive Power Electronics Seminar) consists of Java applets which are embedded in HTML text. iPES is currently available in English, German and Japanese, and shows the basic principles of power electronic systems in form of an introductory university course. A course on advanced power electronics topics is planned for 2002.

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