Modellierung von Kern- und Wicklungsverlusten

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Motivation

Modeling Inductive Components

Employing best state-of-the-art approaches and embedding newly developed approaches into a novel loss calculation framework.
Loss Calculation Framework

Overview

1) **A reluctance model** is introduced to describe the electric / magnetic interface, i.e. $L = f(I)$.

2) **Core losses** are calculated.

3) **Winding losses** are calculated.
Agenda

- Motivation
- Reluctance Model
- Core Losses
- Winding Losses
- Experimental Results
Motivation

Modeling Inductive Components

Calculation of the inductance $L$ with a reluctance model

Inductance calculation

$$L = \frac{N^2}{R_{tot}}$$

Reluctance definition

$$mmf = R_m \Phi$$
Existing Calculation Approaches

Assumption of homogeneous field distribution

\[ R_m = \frac{l_g}{\mu_0 A_g} \]

- \( l_g \): Air gap length
- \( A_g \): Air gap cross-sectional area
Existing Calculation Approaches

Increase of the Air Gap Cross-Sectional Area

Air gap reluctance calculation

\[ R_m = \frac{l_g}{\mu_0 A^*_g} \]

- \( l_g \): Air gap length
- \( A^*_g \): “New” air gap cross-sectional area

\( R_m = \frac{l_g}{\mu_0 (a + l_g)(d + l_g)} \)

e.g. [4] (for a cross section with dimension \( a \times d \)): 
Aim of new model

Air gap reluctance calculation that

- considers the three dimensionality,
- is reasonable easy-to-handle,
- is capable of modeling different shape of air gaps,
- while still achieving a high accuracy.

Illustration of different air gap shapes:
Derivation of new model

**Capacitance-To-Reluctance Analogy**

If air is the dielectric, capacitance $C$ can be expressed as

$$ C = \varepsilon_0 F(g) $$

The reluctance of an air gap with the same geometry is then

$$ R_{m,\text{airgap}} = \frac{1}{\mu_0 F(g)} $$

$$ R_{m,\text{airgap}} = \frac{d}{\mu_0 A} $$
Derivation of new model

The Schwarz-Christoffel Transformation for Air Gaps
Derivation of new model

Basic Structure for the Air Gap Calculation (2-D)

\[
R'_{\text{basic}} = \frac{1}{\mu_0 \left[ \frac{w}{2l} + \frac{2}{\pi} \left( 1 + \ln \frac{\pi h}{4l} \right) \right]}
\]
Derivation of new model

2-D (1)

Basic structure for the air gap calculation:
Derivation of new model

2-D (2)

Air gap type 1

Air gap type 2

Air gap type 3
Derivation of new model

2D $\rightarrow$ 3D : Fringing Factor (1)

Illustrative Example

zy-plane

$R_{zy}^{'}$

zx-plane

$R_{zx}^{'}$
Derivation of new model

2D $\rightarrow$ 3D : Fringing Factor (2)

Illustrative Example

zy-plane

$\sigma_y = \frac{R'_{zy}}{a} \frac{a}{\mu_0 b}$

“Idealized” air gap (no fringing flux)

zx-plane

$\sigma_x = \frac{R'_{zx}}{a} \frac{a}{\mu_0 t}$
Derivation of new model

2D $\rightarrow$ 3D

Fringing Factor

Illustrative Example

3-D Fringing Factor:
\[
\sigma = \sigma_x \sigma_y
\]

\[
R_g = \sigma \frac{a}{\mu_0 b t}
\]

“Idealized” air gap (no fringing flux)
Results

3-D FEM Simulation

Modeled Example

\[ w = 40 \text{ mm}; \ h = 40 \text{ mm} \]
Results

Experimental Results

Inductance Calculation

EPCOS E55/28/21, \( N = 80 \)

<table>
<thead>
<tr>
<th>Air Gap Length</th>
<th>Calculated classically (3)</th>
<th>Calculated with new approach (12)</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 mm</td>
<td>1.42 mH</td>
<td>1.97 mH</td>
<td>2.07 mH</td>
</tr>
<tr>
<td>1.5 mm</td>
<td>0.96 mH</td>
<td>1.47 mH</td>
<td>1.58 mH</td>
</tr>
<tr>
<td>2.0 mm</td>
<td>0.72 mH</td>
<td>1.22 mH</td>
<td>1.26 mH</td>
</tr>
</tbody>
</table>
Results
Experimental Results

Saturation Calculation
EPCOS E55/28/21, N = 80,
\( l_g = 1\) mm, \( B_{\text{sat}} = 0.45\) T

Calculations

<table>
<thead>
<tr>
<th>( L )</th>
<th>Calculated classically (3)</th>
<th>Calculated with new approach (12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.75 mH</td>
<td>3.55 mH</td>
<td></td>
</tr>
<tr>
<td>( I_{\text{sat}} )</td>
<td>4.6 A</td>
<td>3.6 A</td>
</tr>
</tbody>
</table>

Measurement

\( I_{\text{sat}} = 3.7\) A
Agenda

- Motivation
- Reluctance Model
- Core Losses
- Winding Losses
- Experimental Results
Core Losses
Different Modeling Approaches (1)

Steinmetz Equation SE

\[ P = k f^\alpha B^\beta \]

- Only sinusoidal waveforms (\( \rightarrow \) iGSE).

iGSE

\[ P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} dt \]

- DC bias not considered
- Relaxation effect not considered
- Steinmetz parameter are valid only in a limited flux density and frequency range
Core Losses
Different Modeling Approaches (2)

Relaxation Losses

\[ P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} \, dt \]

\[ P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} \, dt + \sum_{l=1}^n Q_{il} P_{il} \]

iGSE

i^2GSE

Loss increase in the zero voltage phases
(where dB/dt = 0)!
Core Losses
Different Modeling Approaches (3)

\[ P_v = \frac{1}{T} \int_0^T k_i \left( \frac{dB}{dt} \right)^\alpha \Delta B^{\beta-\alpha} dt + \sum_{l=1}^n Q_{vl} P_{vl} \]

Remaining Problems

Steinmetz parameter are valid only in a limited flux density and frequency range.

Core Losses vary under DC bias condition.

Modeling relaxation and DC bias effects need parameters that are not given by core material manufacturers.

Measuring core losses is indispensable!
Core Losses
Different Modeling Approaches (4)

Loss Map
(Loss Material Database)

4th dimension: temperature.

Question What loss map structure is needed to take all loss effects into consideration?
Core Losses

Needed Loss Map Structure

Typical flux waveform

Content of Loss Map

- Relaxation
  - $\alpha_r$, $\beta_r$, $k_r$, $\tau$, $\gamma_r$
  - LF

- B-H-Relation
  - HF
Core Losses
Hybrid Loss Modeling Approach (1)

“The best of both worlds”

\[
\begin{align*}
P_v &= \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} dt + \sum_{l=1}^{n} Q_{dl} P_{dl} \\
&\quad + \sum_{l=1}^{n} Q_{dl} P_{dl} \\
&\quad \rightarrow \frac{P}{V}
\end{align*}
\]
Core Losses
Hybrid Loss Modeling Approach (2)
Core Losses
Hybrid Loss Modeling Approach (3)

Loss Map

\[ P_v = \frac{1}{T} \int_0^T k_i \left| \frac{dB}{dt} \right|^\alpha (\Delta B)^{\beta-\alpha} dt + \sum_{l=1}^n Q_{il} P_{vl} \]

\[ P = \frac{P_v}{V} \]

\[ P_{V1} = k_i (2f_1^\alpha) (\Delta B_1/2)^\beta \]
\[ P_{V2} = k_i (2f_2^\alpha) (\Delta B_2/2)^\beta \]
\[ P_{V3} = k_i (2f_3^\alpha) (\Delta B_3/2)^\beta \]
Core Losses
Hybrid Loss Modeling Approach (4)

Interpolation and Extrapolation
\((H_{\text{DC}}^*, T^*, \Delta B^*, f^*)\)

- \(H_{\text{DC}}\) and \(T\)
- \(\Delta B\) and \(f\)
Core Losses
Hybrid Loss Modeling Approach (5)

Advantages of combined approach (loss map and $i^2$GSE):

Relaxation effects are considered ($i^2$GSE).

A good interpolation and extrapolation between premeasured operating points is achieved.

Loss map provides accurate $i^2$GSE parameters for a wide frequency and flux density range.

A DC bias is considered as the loss map stores premeasured operating points at different DC bias levels.
Core Losses
Effect of Core Shape

Procedure

1) The flux density in every core section of (approximately) homogenous flux density is calculated.

2) The losses of each section are calculated.

3) The core losses of each section are then summed-up to obtain the total core losses.
Agenda

- Motivation
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- Winding Losses
- Experimental Results
Winding Losses
Skin Effect in Solid Round Conductors

\[ P_S = F_R(f) \cdot R_{DC} \cdot \hat{I}^2 \quad \text{(Loss per unit length)} \]

with

\[ R_{DC} = \frac{4}{\sigma \pi d^2} \]
\[ \xi = \frac{d}{\sqrt{2} \delta} \]
\[ \delta = \frac{1}{\sqrt{\pi \mu_0 \sigma f}} \]
\[ F_R = \frac{\xi}{4\sqrt{2}} \left[ \frac{\text{ber}_0(\xi)\text{bei}_1(\xi) - \text{ber}_0(\xi)\text{ber}_1(\xi)}{\text{ber}_1(\xi)^2 + \text{bei}_1(\xi)^2} - \frac{\text{bei}_0(\xi)\text{ber}_1(\xi) - \text{bei}_0(\xi)\text{bei}_1(\xi)}{\text{ber}_1(\xi)^2 + \text{bei}_1(\xi)^2} \right] \]
Winding Losses
Proximity Effect in Solid Round Conductors

\[ P_P = G_R \left( f \right) \cdot R_{\text{DC}} \cdot \hat{H}_e^2 \]  
(Loss per unit length)

with

\[ R_{\text{DC}} = \frac{4}{\sigma \pi d^2} \]

\[ \xi = \frac{d}{\sqrt{2} \delta} \]

\[ \delta = \frac{1}{\sqrt{\pi \mu_0 \sigma f}} \]

\[ G_R = -\frac{\xi \pi^2 d^2}{2\sqrt{2}} \left[ \frac{\text{ber}_2(\xi)\text{ber}_1(\xi) + \text{ber}_2(\xi)\text{bei}_1(\xi)}{\text{ber}_0(\xi)^2 + \text{bei}_0(\xi)^2} + \frac{\text{bei}_2(\xi)\text{bei}_1(\xi) - \text{bei}_2(\xi)\text{ber}_1(\xi)}{\text{ber}_0(\xi)^2 + \text{bei}_0(\xi)^2} \right] \]
Winding Losses
Calculation of external field $H_e$ (1D - approach)

Un-gapped cores (e.g. in transformers)

$$P = R_{DC} \left( F_R \hat{I}^2 NM + NG_R \sum_{m=1}^{M} \hat{H}_{avg,m}^2 \right) l_m$$

with

$$H_{avg} = \frac{1}{2} \left( H_{left} + H_{right} \right)$$
Winding Losses
Calculation of external field $H_e$ (2D - approach)

Gapped cores: 2D approach is necessary!
Winding Losses
Effect of the air gap fringing field

The air gap is replaced by a fictitious current, which …

… has the value equal to the magneto-motive force (mmf) across the air gap.
Winding Losses
Effect of the core material

The method of images (mirroring)

“Pushing the walls away”
Winding Losses
Calculation of external field $H_e$ (2D - approach)

Gapped cores: 2D approach

Winding Arrangement

External field vector across conductor $q_{x_iy_k}$

$$
\hat{H}_e = \sum_{u=1}^{m} \sum_{l=1}^{n} \epsilon(u, l) \frac{\hat{i}_{x_u, y_l} ((y_l - y_k) - j(x_u - x_i))}{2\pi ((x_u - x_i)^2 + (y_l - y_k)^2)}
$$
Winding Losses
Different Winding Sections

Section 1: Core window - many mirroring
Section 2: Not core window - one mirroring
Winding Losses
FEM Simulations

Major Simplification

- magnetic field of the induced eddy currents neglected.
- This can be problematic at frequencies above (rule-of-thumb)

\[ f_{\text{max}} = \frac{2.56}{\pi \mu_0 \sigma d^2} \]

Results of considered winding arrangements

<table>
<thead>
<tr>
<th>f-range</th>
<th>( f &lt; f_{\text{max}} )</th>
<th>( f &gt; f_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>&lt; 5%</td>
<td>&gt; 5% (always &lt; 25%)</td>
</tr>
</tbody>
</table>
Agenda

- Motivation
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- Experimental Results
Experimental Results
Measurement 1

Flux Waveform

Inductor
EPCOS E55/28/21, \( N = 18 \),
\( d = 1.7 \text{ mm}, l_g = 1 \text{ mm} \)

Results (measured with power analyzer Yokogawa WT3000)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0.06</td>
<td>0.11</td>
<td>0.17</td>
<td>0.18</td>
<td>-5.56</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0.32</td>
<td>0.42</td>
<td>0.74</td>
<td>0.77</td>
<td>-3.90</td>
</tr>
<tr>
<td>0.25</td>
<td>2</td>
<td>0.5</td>
<td>0</td>
<td>0.14</td>
<td>0.12</td>
<td>0.26</td>
<td>0.27</td>
<td>-3.70</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>0.5</td>
<td>0</td>
<td>0.72</td>
<td>0.49</td>
<td>1.21</td>
<td>1.21</td>
<td>0.00</td>
</tr>
<tr>
<td>0.25</td>
<td>5</td>
<td>0.5</td>
<td>0</td>
<td>0.4</td>
<td>0.23</td>
<td>0.63</td>
<td>0.61</td>
<td>3.28</td>
</tr>
<tr>
<td>0.5</td>
<td>5</td>
<td>0.5</td>
<td>0</td>
<td>2.04</td>
<td>0.92</td>
<td>2.96</td>
<td>2.70</td>
<td>9.63</td>
</tr>
</tbody>
</table>
Experimental Results
Measurement 2

Flux Waveform

Inductor
EPCOS E55/28/21, \( N = 18 \),
\( d = 1.7 \text{ mm}, \ l_g = 1 \text{ mm} \)

\( f_{LF} = 100 \text{ Hz} \)
\( f_{HF} = 10 \text{ kHz} \)

Results (measured with power analyzer Yokogawa WT3000)

<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>0.25</td>
<td>0.15</td>
<td>0.44</td>
<td>0.35</td>
<td>0.79</td>
<td>0.76</td>
<td>3.95</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.5</td>
<td>0.3</td>
<td>1.83</td>
<td>1.51</td>
<td>3.35</td>
<td>3.6</td>
<td>-6.94</td>
</tr>
</tbody>
</table>
Experimental Results 3
Comparison Circuit Simulated Modeling vs. Measurements on a Boost Inductor Employed in a Three-Phase PFC Rectifiers [3]

Results

\[ I_{\text{HF,pp,max}} = 4.1 \, \text{A} \]
\[ P_{\text{Loss}} = 43.3 \, \text{W} \]

(b)

\[ I_{\text{HF,pp,max}} = 4.7 \, \text{A} \]
\[ P_{\text{Loss}} = 46.8 \, \text{W} \]

Photo

Conclusion

Loss modeling very accurate.
Conclusion & Outlook

A high accuracy in modeling inductive components has been achieved.

The following effects have been considered:

**Reluctance model [1]:**
- Air gap stray field.
- Non-linearity of core material considered.

**Core Losses [2]:**
- DC Bias (Loss Map)
- Relaxation Effects ($i^2$GSE)
- Different flux waveforms ($i$GSE / $i^2$GSE)
- Wide range of flux density and frequencies (Loss Map)

**Winding Losses [2]:**
- Skin and proximity effect
- Stray field proximity effect
- Effect of core material
Thank you!

Do you have any questions?
References


