Analysis and Measurement of 3D Torque and Forces for Permanent Magnet Motors with Slotless Windings

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Abstract—Slotless windings, skewed and rhombic, are widely used in industry. Beside the drive torque, possibly undesired transverse torques and forces are generated, which have not been analyzed previously. An analytical derivation of the torque and force components in all three directions is detailed in this paper for the skewed winding. It is shown that for some winding configurations alternating transverse torque components are generated, which may compromise stable operation in applications where for example magnetic or gas bearings are involved. Moreover the windings are analyzed with regard to a potential use as active magnetic radial bearings in high-speed applications. Finally, measurements are presented to verify the theoretical results.

I. INTRODUCTION

Slotless windings are widely used for small sized electric motors in industrial applications ranging from a hundred milliatts to a few hundred watts [1]. The main advantages of a slotless design are the absence of cogging torque, the elimination of losses caused by slot space harmonics, the low winding inductance. For the last two reasons it is also an excellent choice for high-speed motors [2].

An overview of different types of slotless windings is given in [3] and [4]. Commonly used winding types are the skewed type and the rhomic winding. As can be seen from Figure 1 both windings exhibit complex three dimensional structures with two layers overlapping. This can result in undesirable transverse torque and force components beside the intended drive torque in axial direction, even for perfect alignment and symmetry of the winding. In some high-speed applications where rotordynamics must be considered, e.g. when magnetic bearings or gas bearings are involved, these transverse torque components have to be either avoided by choosing an appropriate winding type, configuration and geometry or at least exactly quantified and incorporated into the rotordynamic model.

The magnetic field in the airgap required for the analysis of the torque and force components has been studied in [5], [6] and [7]. Winding factors for the harmonic field components resulting in alternating drive torque have been analyzed in [4]. However, no analysis or measurement of the transverse torque and forces components have been reported in literature.

In this paper torque and force calculations for the skewed type winding are derived. The most common configuration, the two-pole, three-phase winding is analyzed in detail, including possible force generation for an active magnetic bearing. A winding test bench is built and the theoretical results are verified by measurements.

II. DERIVATION OF FORCE AND TORQUE FOR THE SKEWED WINDING

A. Magnetic Field Distribution

According to [5] the flux density in the airgap of a slotless permanent-magnet machine can be described by Fourier series for both the radial and the azimuthal field components as

\[ B_r = \sum_{n=1}^{\infty} B_r^{(n)} \cos(nq\theta + \delta) \]
\[ B_\theta = \sum_{n=1}^{\infty} B_\theta^{(n)} \sin(nq\theta + \delta), \]

where \( q \) is the number of pole pairs of the rotor, \( \delta \) the load angle and \( B_r^{(n)} \) and \( B_\theta^{(n)} \) are the Fourier coefficients of the \( n^{th} \) harmonic. The field in axial direction is assumed to be \( B_z = 0 \).

It is well known that for machines driven with symmetric sinusoidal currents only the fundamental component contributes to a constant torque. Therefore, from a design point of view,
the flux density of an optimal machine only consists of the fundamental component, and hence for the winding analysis carried out in this paper it is assumed that that the geometry of the rotor and the permanent-magnets is chosen such that all the harmonic field components vanish. Indeed for the \( q = 1 \) pole pair machine a purely fundamental field distribution is achieved by a diametral magnetization direction in the cylindrical permanent-magnet [7].

The time varying fundamental field in the airgap for a rotor rotating with speed \( \Omega \) is then described in cylindrical coordinates by

\[
\begin{align*}
B_r &= \hat{B}_r(r) \cdot \cos(q\theta - q\Omega t + \delta) \\
B_\theta &= \hat{B}_\theta(r) \cdot \sin(q\theta - q\Omega t + \delta)
\end{align*}
\]  

To derive the force and torque components in cartesian coordinates is described by

\[
B = \begin{bmatrix}
B_r \cdot \cos(\theta) & -B_\theta \cdot \sin(\theta) \\
B_r \cdot \sin(\theta) & B_\theta \cdot \cos(\theta)
\end{bmatrix}
\]

\[\text{(5)}\]

\[
B = \begin{bmatrix}
B_r \cdot \cos(\theta - \frac{2\pi}{p}) \\
B_\theta \cdot \sin(\theta - \frac{2\pi}{p})
\end{bmatrix}
\]

\[\text{(6)}\]

\[\text{(7)}\]

\[\text{(8)}\]

\[\text{(9)}\]

\[\text{(10)}\]

\[\text{(11)}\]

\[\text{(12)}\]

\[\text{(13)}\]
The torque and the force generated by a single phase is then obtained by integration over one winding loop and one phase zone and summation over all pole pairs as

$$T_{ph} = J \sum_{k=0}^{p-1} \int_{\phi} \int_{r} \int_{\theta} \frac{v \times \frac{\partial w}{\partial \phi} \times B}{2 \sqrt{1 + \left(\frac{\pi r p}{L}\right)^2}} d\theta dr d\phi$$

and

$$F_{ph} = J \sum_{k=0}^{p-1} \int_{\phi} \int_{r} \int_{\theta} \frac{\partial w}{\partial \phi} \times B}{2 \sqrt{1 + \left(\frac{\pi r p}{L}\right)^2}} d\theta dr d\phi,$$

where $R_3$ and $R_4$ are the winding inner and the winding outer radius (see Figure 4). Summing up over all $m$ phases yields the total torque and the total force

$$T = \sum_{k=0}^{m-1} T_{ph} \bigg|_{\phi = \frac{2\pi k}{m}}$$

and

$$F = \sum_{k=0}^{m-1} F_{ph} \bigg|_{\phi = \frac{2\pi k}{m}}.$$

D. Solution for $p = q$

Evaluating (14) and (16) yields the the torque

$$T = \begin{bmatrix} T_x \\ T_y \\ -4m \sin(\frac{\pi}{m})J L \sin(\delta) \end{bmatrix} \int_{R_3}^{R_4} \frac{R_3}{\sqrt{1 + \left(\frac{\pi r p}{L}\right)^2}} d\phi$$

where for $p = 1$ and $m = 3$ the transverse torque components are

$$T_x = \frac{3\sqrt{3}J L^2 \sin(2\Omega t - \delta)}{8\pi} \int_{R_3}^{R_4} \frac{R_3}{\sqrt{1 + \left(\frac{\pi r p}{L}\right)^2}} d\phi$$

and

$$T_y = \frac{3\sqrt{3}J L^2 \cos(2\Omega t - \delta)}{8\pi} \int_{R_3}^{R_4} \frac{R_3}{\sqrt{1 + \left(\frac{\pi r p}{L}\right)^2}} d\phi.$$

For any other combination of $m$ and $p > 1$ the transverse torque components vanish ($T_x = T_y = 0$).

Evaluating (15) and (17) yields a vanishing force $F = 0$ for any combination of $p$ and $m$.

E. Two-Pole Three-Phase Winding

For the two-pole three-phase winding, $q = p = 1$, $m = 3$ the transverse torque components $T_x$ and $T_y$ given by (19) and (20) do not vanish but alternate with a frequency of twice the rotational speed. In order to compare the amplitudes of the drive torque $T_x$ and the transverse torque components $T_x$ and $T_y$ the integral over the radius needs to be solved and therefore the radial dependency of the flux density has to be considered.

The equations for the flux density $B$ for $q = 1$ are derived in [7] and the parameter definitions are given in Figure 4. Simplifying the equations for the radial and azimuthal components of the flux density by setting the relative permeability of the permanent-magnet and the stator back iron $\mu_{r,p,m} \approx 1$ respectively $\mu_{r,F_e} \to \infty$ yields

$$\hat{B}_r = \frac{B_{rem} R_4^2}{2 R_4^2 (R_3^2 - 1)}$$

and

$$\hat{B}_\theta = \frac{B_{rem} R_4^2}{2 R_4^2 (R_3^2 - 1)}.$$

The torque generated by a $m = 3$ phase winding is obtained by evaluating (18), (19) and (20) to

$$T = \begin{bmatrix} 3\sqrt{3}J B_{rem} L^2 R_4^2 \sin(2\Omega t - \delta) \cdot K_1 \\ 3\sqrt{3}J B_{rem} L^2 R_4^2 \cos(2\Omega t - \delta) \cdot K_1 \end{bmatrix} \int_{R_3}^{R_4} \frac{R_3}{\sqrt{1 + \left(\frac{\pi r p}{L}\right)^2}} d\phi$$

where the $K_1$ and $K_2$ are given by

$$K_1 = \log \left( \frac{L + \sqrt{L^2 + \pi^2 R_3^2}}{L + \sqrt{L^2 + \pi^2 R_4^2}} \cdot \frac{L - \sqrt{L^2 + \pi^2 R_3^2}}{L - \sqrt{L^2 + \pi^2 R_4^2}} \right)$$

and

$$K_2 = \frac{1}{R_4^2} \left[ \pi R_4 \sqrt{L^2 + \pi^2 R_3^2} - \pi R_3 \sqrt{L^2 + \pi^2 R_4^2} + (2\pi^2 R_3^2 - L^2) \cdot \log \left( \frac{\pi R_4 + \sqrt{L^2 + \pi^2 R_3^2}}{\pi R_3 + \sqrt{L^2 + \pi^2 R_4^2}} \right) \right].$$

A high ratio of $L/R_4$ and $R_3/R_4$ results in windings producing relatively high transverse torque $T_{xy}$ compared to the drive torque $T_x$ as can be seen from Figure 5. This is rather unfavorable since for common machines the winding length is usually larger than its outer diameter leading to a
the transverse torque in the area of 10% to 20% of the drive torque or more.

III. FORCE GENERATION FOR ACTIVE MAGNETIC BEARINGS

The skewed type winding is a promising candidate for an active magnetic bearing for high-speed applications due to its low inductance and the relatively low negative stiffness resulting from the large magnetic air gap [8]. There are many configurations for generating a net force; e.g. for the configuration \( p = q + 1 \) the torque always vanishes: \( T = 0 \). The force for this configuration and \( m \geq 3 \) is obtained from (15) and (17) as

\[
F = -\frac{2mJL\sin\left(\frac{\pi}{m}\right)}{\pi} \int_{R_3}^{R_4} \left( \tilde{B}_r + \tilde{B}_\theta \right) r dr \left[ \begin{array}{c} \cos(\delta) \\ \sin(\delta) \\ 0 \end{array} \right].
\] (26)

For \( q = 1 \) the simplifications for the flux density \( B \) made in section II-E can be used to calculate the integral in (26) yielding

\[
F = -\frac{m \sin(\pi/m) K_3 JLR_3^2 B_{rem}}{\pi} \left[ \begin{array}{c} \cos(\delta) \\ \sin(\delta) \\ 0 \end{array} \right]
\] (27)

where \( K_3 \) is defined as

\[
K_3 = \log \left( \frac{2L + \sqrt{4L^2 + \pi^2R_3^2}}{2L + \sqrt{4L^2 + \pi^2R_4^2}} \right) \left( \frac{2L - \sqrt{4L^2 + \pi^2R_3^2}}{2L - \sqrt{4L^2 + \pi^2R_4^2}} \right). \] (28)

IV. MEASUREMENTS

A. Measurement Setup

The measurements are performed on a granite measurement platform. The permanent-magnet is mounted on a \( xyz \)-translation stage with an additional rotational stage allowing for accurate positioning and rotation of the magnet. The stator is mounted on a multi component load cell to measure the winding reaction force and torque components resulting from the injected winding currents. A photograph of the described assembly is shown in Figure 6.

A Spitzenberger&Spies DM3000 power supply is used to generate the three-phase symmetric sinusoidal currents fed to the star configured winding at a frequency of 1 Hz. The three phase currents are measured with current sensors (LEM LTS 6-NP). The current sensor output signals and force signals are acquired by a National Instruments LabVIEW card at a sample rate of 1 kHz.

The motor winding to be measured is expected to deliver a nominal drive torque of 30 mNm. In order to measure such small quantities a piezoelectric multi component dynamometer (Kistler 9256C1) is used, providing six force measurements in pairs \( F_{x1} \) and \( F_{x2} \), \( F_{y1} \) and \( F_{y2} \) and \( F_{z1} \) and \( F_{z2} \), and thereby allowing to compute the resulting forces as well as two of the three torque components. The force components are \( F_x = F_{x1} + F_{x2} \), \( F_y = F_{y1} + F_{y2} \) and \( F_z = F_{z1} + F_{z2} \) and the two torque components are \( T_y = a \cdot (F_{x1} - F_{x2}) + b \cdot (F_{z1} - F_{z2}) \) and \( T_z = c \cdot (F_{y1} - F_{y2}) + d \cdot F_x \), where the term \( d \cdot F_x \) accounts for the coordinate transformation from the sensor to the stator coordinate system \((x, y, z) \rightarrow (x, y + d, z)\). \( a, b \) and \( c \) result from the load cell geometry. To obtain the torque \( T_x \) the measurements are repeated for the winding rotated by an angle of 90°.

A multichannel charge amplifier from Kistler of type 5017 is used. For all the measurements the amplifier’s internal second order low-pass filters are used and the cut-off frequency is set to 300 Hz.
To verify correct behavior of the load cell, quasi-static measurements of test forces applied by a dial gauge and test weights were carried out. It was concluded that the resulting forces and the torque $T_y$ was measured with high precision, while for $T_z$ the measured values were 32% lower. Therefore a correction factor had to be introduced for the calculation of $T_z$.

As the generated charge produced by the small forces is low and the charge amplifier operates with high gain, signal drifting needs to be compensated. It is common for piezoelectric force measurements, that drift compensation is done manually by subtracting a linear approximation of the drift considering the beginning and the end of the measurement when no forces are applied. For symmetric sinusoidal three phase currents the resulting force and torque components are supposed to be offset-free. This simplifies drift compensation and the measured data can be processed off-line through a FIR high-pass filter and therewith eliminating the frequency components below 1 Hz. The frequency response of the applied drift compensation filter is shown in Figure 7.

## B. Two-Pole Three-Phase Winding

The geometric data for the $q = p = 1$ test motor is listed in Table I. Sinusoidal three phase currents are injected such that the resulting stator field rotates in mathematically positive direction. Figure 8 shows the measured force and torque components for the rotor position $\theta = 0$ after signal drift compensation. Note that for the rotor fixed at position $\theta = 0$ and the stator field rotating in positive direction the load angle is $\delta = \Omega t$. Measurements are repeated for rotor positions in steps of $10^\circ$. Another series of measurements is furthermore required to obtain the torque component $M_x$ by turning the winding by $90^\circ$. Figure 9 shows the combined resulting force and torque components from these measurements for a load angle $\delta = -\pi/2$, where the maximum drive torque is generated. It can be seen that the transverse torque rotates with twice the rotational speed. For the tested winding the ratio of transverse torque to drive torque is expected to be $T_{xy}/T_z = 0.21$, however the measurement shows a different ratio with 0.32, see Figure 5. Also the force components do not vanish completely. These non-vanishing forces are up to 20% of the torque producing differential forces in the sensor. It can be excluded that these discrepancies result from not perfectly symmetric currents, as the current measurement error is smaller than 1% and the amplitude deviation of the injected currents was less than 1%. The errors however may be caused by misalignment of rotor and stator. The centering of the rotor in the axial direction is satisfactorily achieved by minimizing the non-vanishing forces might be the two layer structure and therewith associated asymmetries of the winding which has not been accounted for in the above modeling.

## C. Active Magnetic Bearing Winding

Measurements have also been performed for a slotless type active magnetic bearing winding with a $p = 2$ pole pair skewed winding and a $q = 1$ pole pair rotor. The geometry is chosen the same as for the one pole pair test motor (Table I), except for the winding pole pair number. The measurement procedure
amplitude for a peak current of 1 A resulting in a current density of \( J = 1.43 \text{ A/mm}^2 \) is 0.72 N, while the measured value range is 0.69 N to 0.71 N.

V. Conclusion

The skewed type winding with three phases and one pole pair \( q = p = 1 \) is widely used in industrial motor applications. Despite its popularity, its peculiarity to produce transverse torque components in addition to the drive torque seems not to be well known. However when rotor dynamics are of concern, e.g. when magnetic or gas bearings are involved, a thorough evaluation of the winding behavior is crucial. Therefore analytical expressions for the 3D force and torque are presented in this paper. For the two-pole winding, the measurements verify the existence of transverse torques with twice the frequency of the rotational speed, however with even a higher transverse to axial torque ratio than calculated. For the magnetic bearing winding the theoretical results are in very good agreement with the measurements. This proves the feasibility of using such a winding for a magnetic bearing.

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References