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MINIMIZATION OF THE HARMONIC RMS CONTENT OF THE MAINS CURRENT OF A PWM CONVERTER SYSTEM BASED ON THE SOLUTION OF AN EXTREME VALUE PROBLEM

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Abstract

This paper treats the analytically closed optimization of the modulation method of a DC voltage link PWM converter system. In this modulation method the amplitude is defined, as extension of a simple sinusoidal modulation, by addition of a third harmonic with adjustable amplitiude. The quality functional is given by the sum of the squares of the rms values of the resulting current harmonics, or the harmonic power loss, respectively.

The application of space vector calculus, the assumption of high pulse rate of the converter system and the definition of local mean values (related to a pulse period) make a sufficiently exact, direct and analytically closed formulation of the harmonic power loss possible. This formulation is made in the time domain in dependency on the freely adjustable parameter of the underlying phase modulation functions. A minimization can then be performed via solving a simple extreme value problem without having to use a digital computer. Besides minimum harmonic power loss also the case of maximization of the voltage utilization of the converter system is analyzed.

The calculation method presented verifies classical pulse pattern optimizations (based on digital computer utilization) known from literature which are restricted to converter systems of low pulse rates. Due to the complete avoidance of numerical calculation steps the description of the system behavior can be given in this case directly with the help of simple functional dependencies. Thereby the theory of the stationary operational behavior of PWM converter systems with high pulse rates is substantially extended and, also, an immediate inclusion of the calculation results into the dimensioning of PWM converter systems is made possible.

KEYWORDS: PWM Rectifier/Inverter System, Switching Strategy Optimization, Space Vector Calculus, Harmonic Power Loss

1 Introduction

The control signals of the bridge legs of a voltage DC link PWM converter system (see Fig.1) are defined in the simplest case by the intersection of purely sinusoidal phase modulation functions

$$\begin{aligned} m_R(\varphi_U) &= M\cos\varphi_U \\ m_S(\varphi_U) &= M\cos\left(\varphi_U - \frac{2\pi}{3}\right) \\ m_T(\varphi_U) &= M\cos\left(\varphi_U + \frac{2\pi}{3}\right) \end{aligned} \qquad (1)$$

$$M = \frac{2\hat{U}_{U,(1)}}{U_{ZK}} = \frac{2\hat{U}_U^*}{U_{ZK}} \qquad (2)$$

$$m_{(RST)}(\varphi_U) = \frac{2}{U_{ZK}}\,u_{U,(RST)}(\varphi_U) \qquad m_{(RST)} \in [-1,+1] \qquad (3)$$

with a triangular signal with pulse frequency. According to Fig.2 therefore we have for the relative on-time of the bridge legs (which can be replaced regarding their function by a double-pole switch between positive and negative DC link voltage bus)

$$\begin{aligned} \alpha_R(\tau) &= \frac{2t_{\mu,2}}{T_P} = \frac{1}{2}[1+m_R(\tau)] = \frac{1}{2}+\frac{M}{2}\cos\omega_N\tau \\ \alpha_S(\tau) &= \frac{2t_{\mu,3}}{T_P} = \frac{1}{2}[1+m_S(\tau)] = \frac{1}{2}+\frac{M}{2}\cos\left(\omega_N\tau - \frac{2\pi}{3}\right) \\ \alpha_T(\tau) &= \frac{2t_{\mu,1}}{T_P} = \frac{1}{2}[1+m_T(\tau)] = \frac{1}{2}+\frac{M}{2}\cos\left(\omega_N\tau + \frac{2\pi}{3}\right) \end{aligned} \qquad (4)$$

with

$$\varphi_U = \omega_N\tau\ . \qquad (5)$$

By introduction of space vector calculus

$$\underline{u}_U = \frac{2}{3}U_{ZK}\left[u_{U,R} + \underline{a}\,u_{U,S} + \underline{a}^2\,u_{U,T}\right] \qquad \underline{a} = \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) \qquad (6)$$

we have for weighting of the space vectors (which are associated to the switching states of the PWM converter, see Fig.3)

$$\begin{aligned} \delta_6 &= (\alpha_R - \alpha_T) = \frac{\sqrt{3}M}{2}\sin\left(\varphi_U + \frac{\pi}{3}\right) \\ \delta_2 &= (\alpha_S - \alpha_R) = \frac{\sqrt{3}M}{2}\sin\left(\varphi_U - \frac{\pi}{3}\right) \end{aligned} \qquad (7)$$

$$\delta_0 + \delta_7 = 1 - (\alpha_S - \alpha_T) = 1 - \frac{\sqrt{3}M}{2}\sin\varphi_U \qquad (8)$$

$$\begin{aligned} \delta_7 &= \alpha_T \\ \delta_0 &= (1-\alpha_S) \end{aligned} \qquad (9)$$

(the indices denote the converter switching state by the decimal

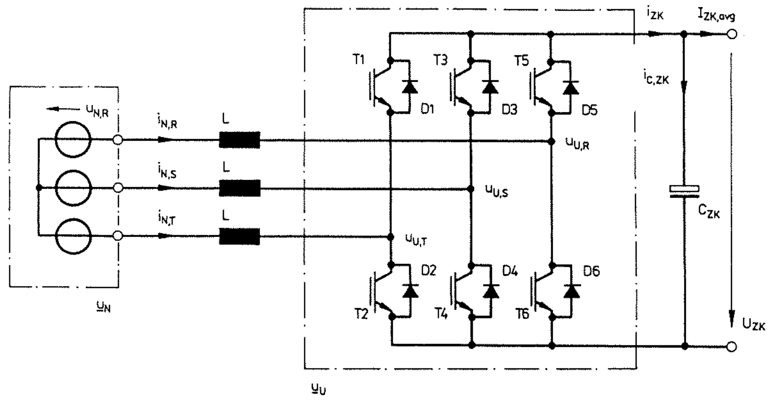


Fig.1: Structure of the power circuit of a three phase voltage DC link PWM converter system. For usage as *PWM inverter* for AC machine drives the inductances L and the three phase system $\underline{u}_N$ can be interpreted as simple equivalent circuit of the AC machine formed by leakage inductances and machine counter emf. On the other hand, for mains operation of the *PWM converter* (PWM rectifier, static VAR compensator) the inductances have to be connected in series; the voltage system $\underline{u}_N$ is defined by the mains conditions.

equivalent of the converter switching status vector interpreted as binary number). The further equations describing the voltage generation are summarized in Appendix 1. According to Eqs.(1) and (3) there follows for the modulation range for sinusoidal modulation

$$M \in [0,1] \ . \tag{10}$$

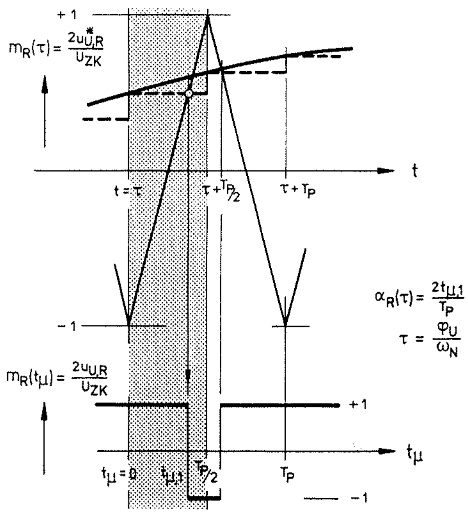
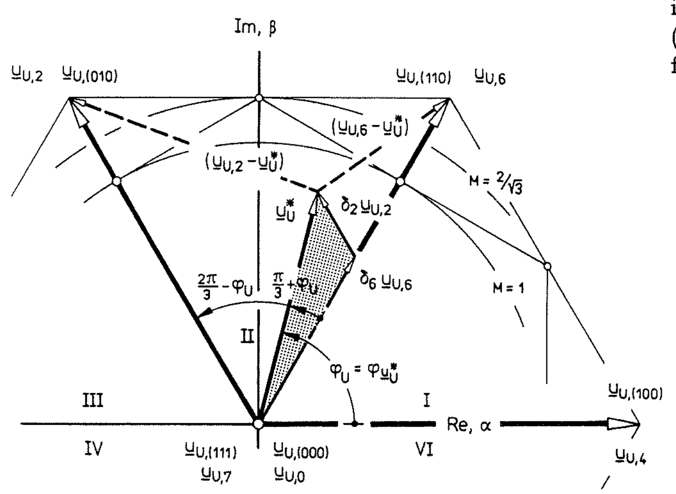


Fig.2: Derivation of the switching times of an converter bridge leg via intersection of the relevant phase modulation function with a triangular signal.



As is shown in Ref.[1], now the modulation limit can be increased by a simple modification of the phase modulation functions

$$\begin{aligned} m_R(\varphi_U) &= M_1 \cos\varphi_U - M_3 \cos 3\varphi_U \\ m_S(\varphi_U) &= M_1 \cos\left(\varphi_U - \frac{2\pi}{3}\right) - M_3 \cos 3\varphi_U \\ m_T(\varphi_U) &= M_1 \cos\left(\varphi_U + \frac{2\pi}{3}\right) - M_3 \cos 3\varphi_U \end{aligned} \tag{11}$$

$$M_1 = \frac{2\hat{U}^*_{U,(1)}}{U_{ZK}} \qquad M_3 = \frac{2\hat{U}^*_{U,(3)}}{U_{ZK}} \tag{12}$$

with

$$\frac{M_3}{M_1} = \frac{1}{6} \tag{13}$$

to the theoretically maximum value

$$M_1 \in \left[0, \frac{2}{\sqrt{3}}\right] \tag{14}$$

(cf. Fig.3).

If the square of the sum of the rms values of the existing current harmonics

$$I = \Delta I^2_{N,RST,rms} = \frac{1}{T_N}\int_{T_N} \Delta i^2_{N,RST,rms}(\tau)\, d\tau \rightarrow Min \tag{15}$$

with

$$\Delta\underline{i}_N = \underline{i}_N - \underline{i}^*_N \tag{16}$$

and

$$\underline{i}^*_N(\tau) = \hat{\underline{I}}^*_N \exp j\omega_N\tau \tag{17}$$

is chosen as quality criterion for the optimization, there follows (cf. Ref.[2]) as suboptimal solution (in the region of low pulse frequencies treated) again the extension of the simple sinusoidal

Fig.3: Approximation of the reference value of the converter output voltage via neighbouring converter voltage space vectors (due to the 60°–symmetry of the voltage space vectors following for the converter switching states one can limit the considerations to the interval of $\varphi_U \in [\frac{\pi}{3}, \frac{2\pi}{3}]$ shown here).

modulation as given in Eq.(11). However, then

$$\frac{M_3}{M_1}\Big|_{I=min} = \frac{1}{4} \tag{18}$$

is valid (cf. Fig.4).

For the optimization of the modulation approach of a PWM converter system regarding maximization of the modulation limit (or minimization of the harmonic loss, respectively) therefore the modification of the simple sinusoidal modulation by adding a third harmonic is of paramount importance. This is analyzed in this paper in greater detail for PWM converter systems for *higher* pulse rates. The calculation is performed in an analytically closed approach as opposed to the methods given in literature which are based on digital computer application (e.g., Ref.[3]). The avoidance of numerical calculation steps is made possible by application of approximations which are valid for high pulse rates with sufficient accuracy. The comparison of the results (which is omitted here for the sake of brevity) to the digital computer approach shows an excellent consistency already for relatively low pulse rates. The importance of the calculation method presented here lies especially in the determination of the functional relationships between the characteristic system quantities and in the associated very much deeper insight into the "inner" system behavior.

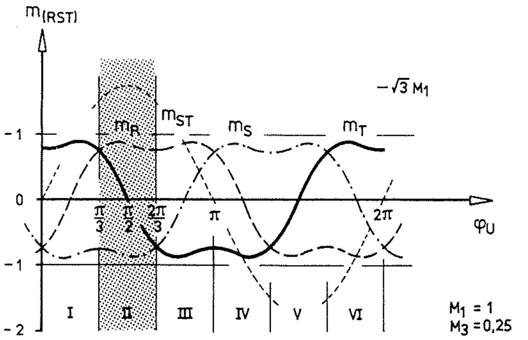


Fig.4: Shape of the phase modulation functions according to Eq.(54) or [3], respectively.

2 Basic Considerations Regarding the Optimizability of the Modulation Method by Modification of Sinusoidal Phase Modulation Functions

As a closer investigation shows there follows via Eqs.(7),(8) or Eq.(9), respectively, that the modification of the phase modulation functions according to Eq.(11) influences only the distribution of the two not voltage forming free-wheeling states (δ_0, δ_7) between begin and end of each pulse half period. This distribution of the free-wheeling status in general gives in the region of high pulse rates the only parameter of the modulation method which can be chosen freely. This is due to the fact that weighting of the voltage forming converter switching states is defined directly via the converter space vector to be generated (cf. Fig.3). Accordingly, in the space vector representation the addition of

$$m_{3,(RST)}(\varphi_U) = -M_3 \cos 3\varphi_U \tag{19}$$

can be interpreted as a special form of a zero-quantity

$$m_0 = \frac{1}{3}\left(m_R + m_S + m_T\right) \tag{20}$$

with

$$\begin{aligned} m_R &= m_R' + m_0 \\ m_S &= m_S' + m_0 \\ m_T &= m_T' + m_0 \end{aligned} \tag{21}$$

(22)

The optimization of the modulation function therefore has to be performed via an appropriate modification of the free-wheeling state distribution mentioned before by giving an appropriate amplitude M_3.

3 Minimization of the Harmonic Losses

The quality criterion shall be defined as the sum of the squares of the rms values of the existing current harmonics or, as harmonic losses, respectively (cf.Eq.(15)). This suggests itself because then there (if the skin effect is neglected) the copper losses as well as the torque pulsations (in the case of supplying to an AC motor where a sinusoidal air gap flux is assumed) are rated. The minimization of the quality criterion assumes the knowledge of the fuctional dependency of the harmonic losses

$$\Delta I^2_{N,RST,rms} = \frac{3}{\pi}\int_{\pi/3}^{2\pi/3}\Big\{\Delta i^2_{N,RST,rms,1}(M_1, M_3, \varphi_U) + \\ +\Delta i^2_{N,RST,rms,2}(M_1,\varphi_U)\Big|_{M_1,M_3=const}\Big\}\, d\varphi_U \tag{23}$$

on the distribution (as mentioned before) of the not voltage forming states (or, equivalent, of the duration of *one* of the two free-wheeling states (cf. Eqs. (9) and (11))). As shown in more detail in Appendix 2 this functional dependency follows via

$$\frac{1}{\Delta i_n^2}\Delta i^2_{N,RST,rms,1} = \\ \frac{48}{9}\delta_7\Big\{\delta_2\delta_6(\delta_2-\delta_6) - 2\delta_{26}\left[1-(\delta_7+\delta_6+\delta_2)\right]\Big\} \tag{24}$$

$$\frac{1}{\Delta i_n^2}\Delta i^2_{N,RST,rms,2} = \\ \frac{16}{9}\Big\{6\delta_{26}^2 + 2\delta_{26} - 4\delta_2^4 - 4\delta_6^4 - 4\delta_2\delta_{26} - 4\delta_6\delta_{26} - \\ - 8\delta_2\delta_6\delta_{26} + 2\delta_2\delta_6^2 - \delta_2^2\delta_6 + 3\delta_2^3\delta_6 - \delta_2^2\delta_6^2\Big\} \tag{25}$$

with

$$\Delta i_n = \frac{U_{ZK}T_P}{8L} \tag{26}$$

and

$$\delta_{26} = \delta_2^2 + \delta_2\delta_6 + \delta_6^2\ , \tag{27}$$

where

$$\begin{aligned} \Delta i^2_{N,RST,rms}(\tau) &= \Delta i^2_{N,RST,rms,1}\{\delta_7(\tau),\delta_6(\tau),\delta_2(\tau)\} + \\ &+ \Delta i^2_{N,RST,rms,2}\{\delta_6(\tau),\delta_2(\tau)\} \end{aligned} \tag{28}$$

or

$$\begin{aligned} \Delta i^2_{N,RST,rms,1} &= \Delta i^2_{N,RST,rms,11}(\delta_{7,1};\delta_2;\delta_6) + \\ &+ \Delta i^2_{N,RST,rms,12}(\delta_{7,1};\delta_{7,2};\delta_2;\delta_6) \end{aligned} \tag{29}$$

with

$$\delta_7 = \delta_{7,1} + \delta_{7,2} = \alpha_T \quad (30)$$

$$\delta_{7,1} = \frac{1}{2} + \frac{M_1}{2}\cos\left(\varphi_U + \frac{2\pi}{3}\right)$$

$$\delta_{7,2} = -\frac{M_3}{2}\cos 3\varphi_U \quad (31)$$

respectively, is valid. The optimization therefore can be limited to that loss contribution

$$\Delta I^2_{N,RST,rms,12} = \frac{3}{\pi}\int_{\pi/3}^{2\pi/3} \Delta i^2_{N,RST,rms,12}(\varphi_U)\,\varphi_U \quad (32)$$

which is influenced via modification of the free-wheeling state distribution by a third harmonic (cf. Eqs.(30), (31) and (29)). The quality criterion (cf. Eq.(15)) therefore becomes

$$I' = \Delta I^2_{N,RST,rms,12} \rightarrow Min\ . \quad (33)$$

Via

$$\frac{1}{\Delta i_n^2}\,\Delta i^2_{N,RST,rms,12} = -\frac{3}{4}M_1^4\frac{M_3}{M_1}\left(1-\frac{2M_3}{M_1}\right)\cos 3\varphi_U \quad (34)$$

there follows

$$\frac{1}{\Delta i_n^2}\,\Delta I^2_{N,rms,12} = -\frac{1}{4}M_1^4\frac{M_3}{M_1}\left(1-\frac{2M_3}{M_1}\right)\frac{3}{\pi}\int_{\pi/3}^{2\pi/3}\cos^2 3\varphi_U\,d\varphi_U\ . \quad (35)$$

The minimum value problem regarding the harmonic losses therefore is reduced to a simple extreme value problem according to

$$\frac{d}{dM_3}\left\{\Delta I^2_{N,RST,rms,12}\right\}\Big|_{M_1=const} = 0\ . \quad (36)$$

This yields with

$$M_{3,I=Min} = \frac{M_1}{4} \quad (37)$$

a result already known from Ref.[2]. The modulation method (in the following designated by subscript [3]) which is suboptimal for the lower pulse rate region (cf. Ref.[2]) therefore becomes the exact solution for the optimization problem for high pulse rates.

A minor limitation of the optimization approach is given, however, due to a reduction of the maximum modulation depth according to

$$M_{1,max}\Big|_{\frac{M_3}{M_1}=\frac{1}{4}} = 1.122 = 0.972\frac{2}{\sqrt{3}}\ . \quad (38)$$

However, this value lies significantly above the value which can be reached with the simple sinusoidal modulation.

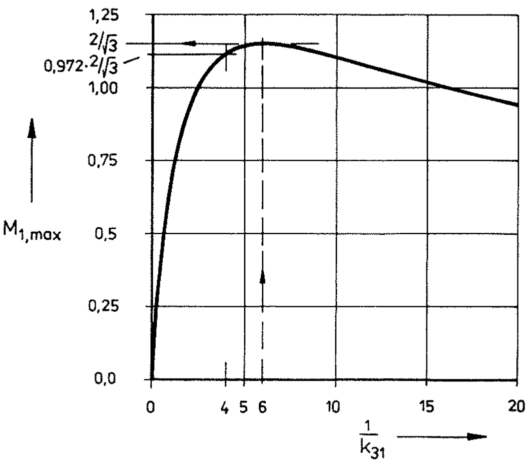


Fig.5: Maximum modulation index $M_{1,max}$ of the converter system in dependency on the modification of the phase modulation functions characterized by k_{31}.

4 Maximization of the Modulation Range

Alternatively to the minimization of the harmonic losses one can optimize (according to Ref.[1]) the modulation method for obtaining the maximum possible modulation range.

By considering the modulation function of phase R there follows the operating region which does not show overmodulation according to

$$\xi = M_1\cos\varphi_U - M_3\cos 3\varphi_U \leq 1 \quad (39)$$

or with

$$\frac{M_3}{M_1} = k_{31}\ , \quad (40)$$

respectively, as

$$\xi = M_1\cos\varphi_U - k_{31}M_1\cos 3\varphi_U \leq 1\ . \quad (41)$$

For the position of the resulting maximum of the modulation function within a fundamental period there follows for the set of parameters M_1, k_{31}

$$\sin^2\varphi_{U,\xi_{max}} = \frac{9k_{31}-1}{12k_{31}} \quad (42)$$

with

$$\frac{d\xi}{d\varphi_U}\Big|_{M_1,k_{31}=const} = 0\ . \quad (43)$$

For the value of the maximum there follows

$$\xi_{max} = M_1 k_{31}\sqrt{1+\frac{1}{3k_{31}}}\left(1+\frac{1}{3k_{31}}\right)\ . \quad (44)$$

At the modulation limit $\xi_{max} = 1$ the modulation index of the fundamental becomes a function

$$M_1 = \frac{1}{k_{31}\left(1+\frac{1}{3k_{31}}\right)^{3/2}} \quad (45)$$

of the parameter k_{31} (cf. Eqs.(40) and (11) or Fig.5, respectively) which can be set. The maximum modulation range therefore is given according to the solution of a extreme value problem via

$$\frac{dM_1}{dk_{31}} = 0 \quad (46)$$

for

$$k_{31,M_1max} = \frac{1}{6}\ . \quad (47)$$

The modulation depth in this case reaches the theoretically possible maximum value

$$M_{1,max}\Big|_{k_{31}=\frac{1}{6}} = \frac{2}{\sqrt{3}} \quad (48)$$

for

$$\varphi_{U,M_1max}\Big|_{k_{31}=\frac{1}{6}} = \frac{\pi}{6}\ . \quad (49)$$

5 Alternate Possibilities for Optimization

In general the minimization of the rms value of the harmonics of a PWM converter system constitutes a variational problem. For the quality functional we have there Eq.(15). According to the relationship Eq.(28) an optimization has to be performed "locally", i.e. within each pulse period via appropriate setting of the free-wheeling state distribution (cf. Fig.6 and Fig.7). It results in the determination of the optimal distribution function

$$\frac{\delta_7}{\delta_0} = \frac{\delta_7}{\delta_0}(\varphi_U) \ . \tag{50}$$

This distribution function can be given according to Eqs.(8), (30) and (31) via the duration of one of the two free-wheeling states (e.g., as in this paper, via δ_7). The associated phase modulation functions follow with Eqs. (7),(9) or (4), respectively.

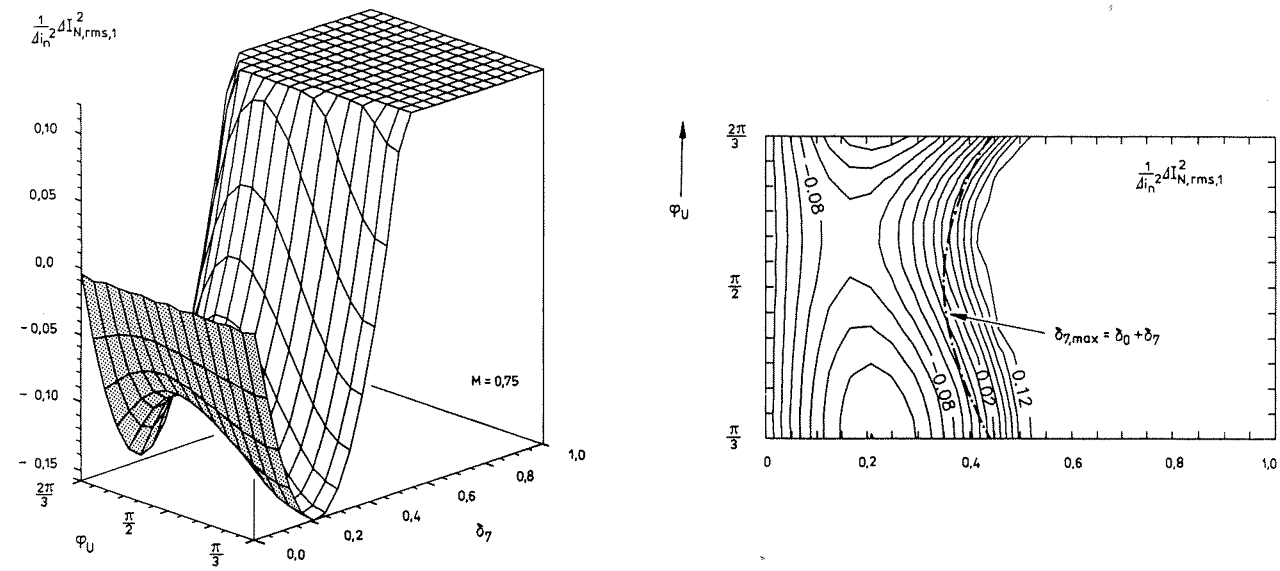


Fig.6: Illustration of the optimizability of the local harmonic power loss, related to a pulse period (cf. Eq.(24)). Dependency of $1/\Delta i_n^2 \ \Delta i^2_{N,RST,rms,1} = 1/\Delta i_n^2 \ \Delta i^2_{N,RST,rms,1}(\delta_7, \varphi_U)$ (M=0.75).

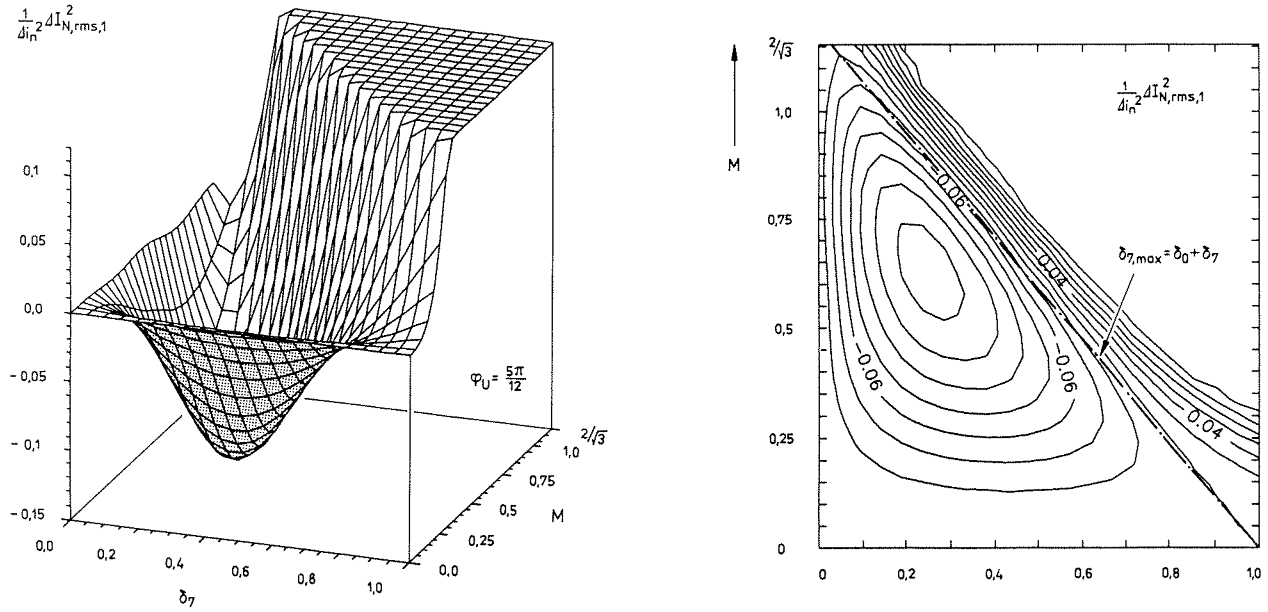


Fig.7: Illustration of the optimizability of the local harmonic power loss, related to a pulse period (cf. Eq.(24)). Dependency of $1/\Delta i_n^2 \ \Delta i^2_{N,RST,rms,1} = 1/\Delta i_n^2 \ \Delta i^2_{N,RST,rms,1}(\delta_7, M)$ ($\varphi_U = 75°$).

With the set-up according to Eq.(11) the structure of the distribution function has been anticipated, however (cf. section 2). The variational problem thereby degenerates to a simple extreme value problem or to an optimization of a quality *function*, respectively, because as the only parameter which can be set freely there remains M_3. The distribution of the free-wheeling state defined herewith not necessarily constitutes the optimal distribution for each pulse interval. Therefore in the following we want to discuss briefly the solution of the variational problem mentioned.

As shown in Ref.[4] or Ref.[5], respectively, there follows

$$\delta_{7,I=Min} = \{\frac{1}{2}[1-(\delta_2+\delta_6)] - \frac{\delta_2\delta_6}{4\delta_{26}}(\delta_2-\delta_6)\} \,. \qquad (51)$$

A simple check leads to

$$\delta_{7,I=Min} = \frac{1}{2}[1+m_T(\varphi_U)] \equiv \delta_{7,[3]} \qquad (52)$$

with

$$m_T(\varphi_U) = M_1 \cos\left(\varphi_U + \frac{2\pi}{3}\right) - \frac{M_1}{4}\cos 3\varphi_U \qquad (53)$$

and

$$\delta_{7,[3]} = \left[\frac{1}{2} + \frac{M_1}{2}\cos\left(\varphi_U + \frac{2\pi}{3}\right) - \frac{M_3}{2}\cos 3\varphi_U\right]_{M_3=\frac{M_1}{4}} \,, \qquad (54)$$

however (cf. Eqs.(9), (11) and (37)). **Thereby local (related to one pulse period) and global (related to one fundamental period) optimization lead to identical results!** The modification calculated in section 3 regarding purely sinusoidal phase modulation functions therefore implies directly also the minimal harmonic power loss contributions of each pulse interval.

As is immediately clear one can give

$$\delta_{7,[2]} = \frac{1}{2}[1-(\delta_2+\delta_6)] = \frac{1}{2}(\delta_0+\delta_7) = \frac{1}{2} - \frac{\sqrt{3}M}{4}\sin\varphi_U \qquad (55)$$

as simple suboptimal approximation (cf. Ref.[6] or Fig.8, respectively). This is of interest due to the then possibly less complex modulation unit for the converter because there the free-wheeling state can simply be distributed in equal parts at the begin and end of each pulse half interval.

6 Comparison of the Results

As mentioned already for PWM converter systems with high pulse rates (independently of the quality functional selected) the only parameter accessible for the optimization of the modulation method is given via the distribution of the not voltage forming state. Accordingly the modulation methods treated here shall be illustrated graphically via the then given free-wheeling state distribution or the duration of one of the two free-wheeling states, respectively. The sum of the duration of the two free-wheeling states occurring within one pulse half period is determined directly via the PWM converter output voltage to be generated (cf. Eq.(8)) and constitutes a side condition of the optimization. In the case of simple sinusoidal modulation (designated by subscript [1]) there follows for constant modulation index a shape

$$\delta_{7,r} = \frac{\delta_7}{\delta_0+\delta_7} \qquad (56)$$

which depends heavily on the angle φ_U (see Fig.9). This is substantially smoothened for the harmonic-optimal modulation method as calculated in section 3 (designated by subscript [3]). A free-wheeling state distribution

$$\delta_{7,r} \approx 0.5 \qquad (57)$$

in this case therefore can be seen as characteristic for a modulation method being optimized with respect to the harmonic power losses. With this there follows immediately that an equal distribution

$$\delta_{7,r} = 0.5 \qquad (58)$$

for the entire angle and modulation range leads to a suboptimal modulation approach (designated by subscript [2]) as defined via Eq.(55).

A comparison of the harmonic power losses in one phase resulting for the modulation functions treated is shown in Fig.10. The equations underlying these characteristics

$$\Delta I^2_{N,rms,[1]} = \frac{1}{6}\Delta i_n^2 M^2\left[1 - \frac{8M}{\sqrt{3}\pi} + \frac{3M^2}{4}\right] \qquad M \in [0,1] \qquad (59)$$

$$\Delta I^2_{N,rms,[2]} = \frac{1}{6}\Delta i_n^2 M^2\left[1 - \frac{8M}{\sqrt{3}\pi} + \frac{9M^2}{8}\left(1 - \frac{3\sqrt{3}}{4\pi}\right)\right] \qquad M \in \left[0, \frac{2}{\sqrt{3}}\right] \qquad (60)$$

$$\Delta I^2_{N,rms,[3]} = \frac{1}{6}\Delta i_n^2 M_1^2\left\{1 - \frac{8M_1}{\sqrt{3}\pi} + \frac{3M_1^2}{4}\left[1 - k_{31}\left(1-2k_{31}\right)\right]\right\} \qquad k_{31} = \frac{1}{4} \qquad M \in \left[0,\ .972\frac{2}{\sqrt{3}}\right] \,, \qquad (61)$$

can be derived in a simple procedure via evaluation of Eq.(15) considering Eqs.(24) and (25) (cf. also Ref.[5]). The harmonic-

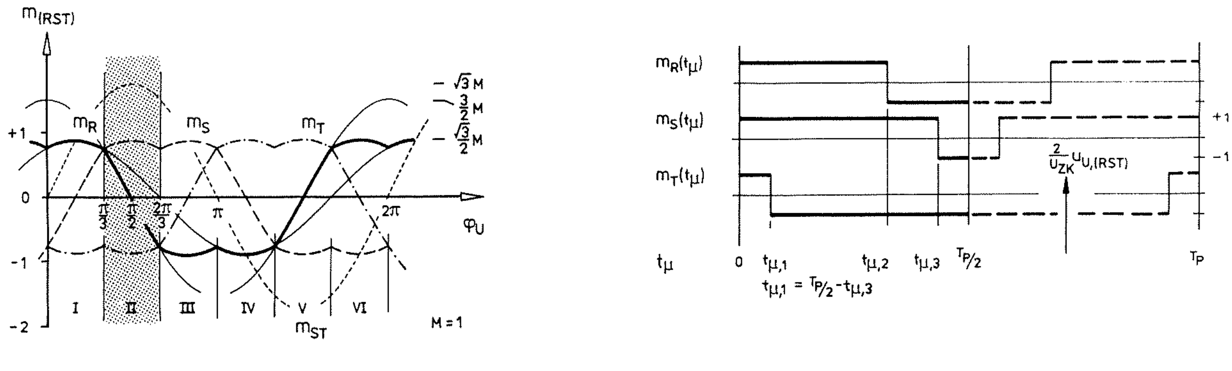


Fig.8: "Microscopic" and "macroscopic" shape of the phase modulation functions according to Eq.(55) or [2], respectively.

optimal modulation method shows especially in the upper modulation region a significant reduction of the harmonic power loss as compared to sinusoidal modulation. The suboptimal solution lies (according to the excellent approximation of the optimal free-wheeling state distribution (cf. Fig.9) very close to the global optimum. In Fig.11 the dependency of the harmonic power loss on the relative amplitude k_{31} of the third harmonic (related to the fundamental) is shown. Due to the relatively flat minimum also the solution gained by maximization of the modulation range (cf. Eq.(47)) leads to very good results.

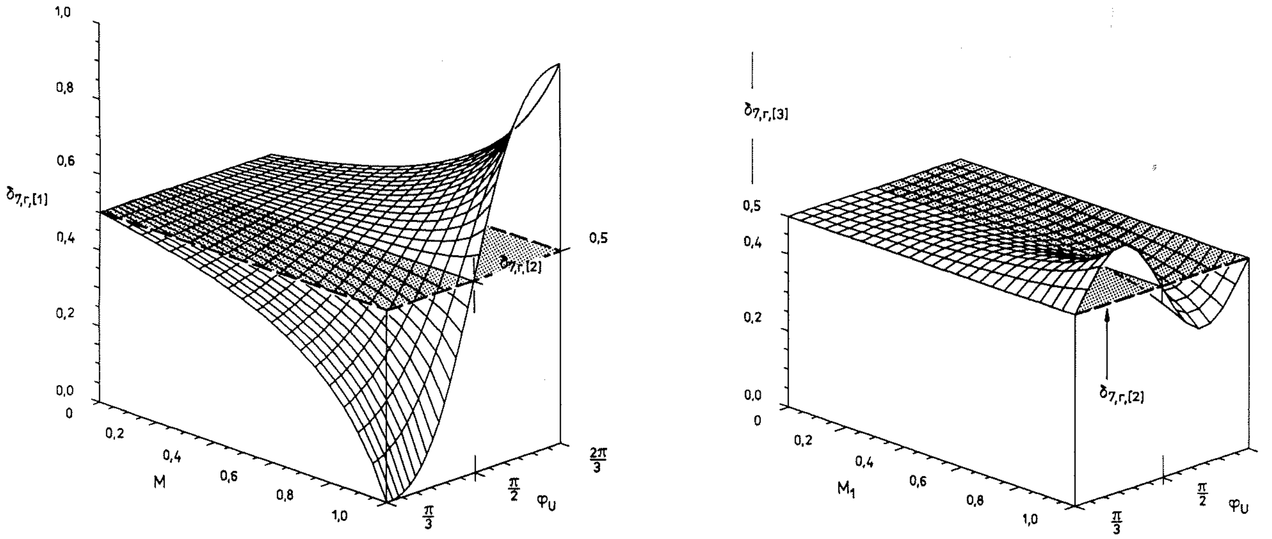


Fig.9: Dependency of the relativ free-wheeling state duration $\delta_{7,r}$ on the modulation index and on the converter voltage angle φ_U; left: sinusoidal modulation; right: harmonic-optimal modulation method.

7 Conclusions

In this paper modulation functions have been analyzed leading to an optimal or suboptimal, respectively, distribution of the free-wheeling states between begin and end of a pulse half period and therefore to a complete switching state sequence

$$\ldots 0\,2\,6\,7\Big|_{t_\mu=0} 7\,6\,2\,0\Big|_{t_\mu=T_P/2} 0\,2\,6\,7\ldots \qquad \varphi_U \in \left[\frac{\pi}{3},\frac{2\pi}{3}\right] . \quad (62)$$

For this class of modulation functions ("continuous" modulation – see Ref.[5]) there has been derived in section 3 a modulation method being optimal with respect to local and global harmonic losses, limited to converter systems with high pulse rate.

If the entire free-wheeling state is shifted to one end of a pulse half period there follows a further group of modulation functions ("discontinuous" modulation) which is treated in detail in Ref.[5]. It can be shown that for these modulation methods in the upper modulation range a substantional reduction of the harmonic losses is given as compared to continuous modulation (cf. Fig.10). In this paper a more detailed treatment has to be omitted for the sake of brevity, however.

In conclusion it shall be pointed out that the approximation methods giving the basis of the optimization performed here can be very successfully applied also to the analysis of other power electronic systems, e.g., direct AC-AC PWM converters. In each case a clear representation for an engineering point of view is supported. This is due to the functional dependencies of the characteristic system quantities determined and opposed to purely nu-

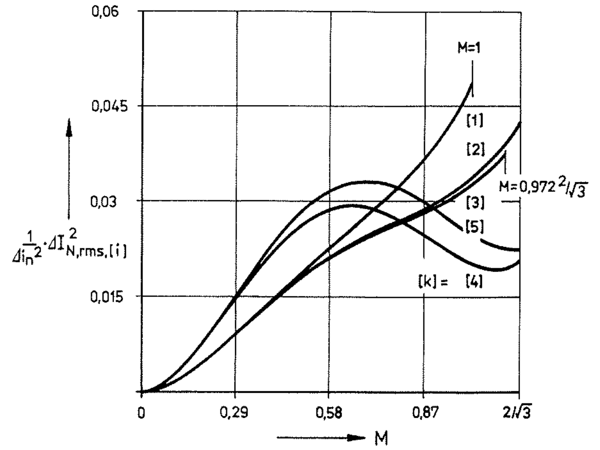


Fig.10: Comparison of the normalized harmonic power losses for various modulation methods ([1]: Sinusoidal modulation (Eq.(59)); [2]: Suboptimal space vector modulation (Eq.(60)); [3]: Local and global optimal sinusoidal modulation with added third harmonic ($M_3 = M_1/4$, verified in sections 3 or 5, respectively; Eq.(61)); [4],[5]: Optimal control methods derived in Ref.[5]).

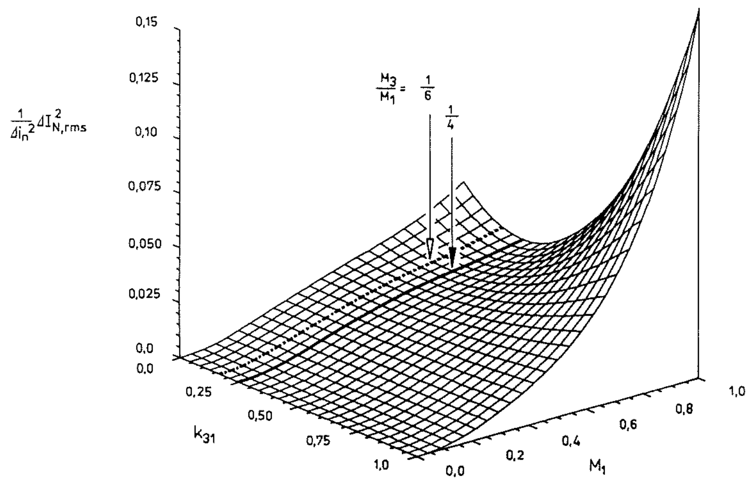


Fig.11: Global harmonic losses of one phase in dependency on the fundamental amplitude and on the ratio $k_{31} = M_3/M_1$ (cf. Eq.(61)).

merical calculation methods. Furthermore, dimensioning of the system components is simplified.

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Appendix A

According to Fig.3 we have:

$$\frac{\delta_6}{\delta_2} = \frac{\sin(\frac{2\pi}{3} - \varphi_U)}{\sin(\varphi_U - \frac{\pi}{3})} \qquad \varphi_U \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right] \tag{A.1}$$

$$\underline{u}_U^*(\tau) = \frac{2}{T_P}\left[\int_{t_{\mu,1}(\tau)}^{t_{\mu,2}(\tau)} \underline{u}_{U,6}\, dt_\mu + \int_{t_{\mu,2}(\tau)}^{t_{\mu,3}(\tau)} \underline{u}_{U,2}\, dt_\mu\right] \tag{A.2}$$

$$\underline{u}_U^* = \underline{\hat{U}}_U^* \exp j\omega_N \tau \tag{A.3}$$

$$\begin{aligned} \underline{u}_{U,6} &= -\underline{a}^2 \frac{2}{3} U_{ZK} \\ \underline{u}_{U,2} &= +\underline{a} \frac{2}{3} U_{ZK} \end{aligned} \tag{A.4}$$

$$\delta_6 = \frac{2}{T_P}\left[t_{\mu,2} - t_{\mu,1}\right] \qquad \delta_2 = \frac{2}{T_P}\left[t_{\mu,3} - t_{\mu,2}\right] \tag{A.5}$$

$$\underline{u}_U^*(\varphi_U) = \underline{u}_{U,6}\,\delta_6 + \underline{u}_{U,2}\,\delta_2 \qquad \varphi_U = \omega_N \tau \tag{A.6}$$

$$u_U^*(\varphi_U) = \frac{2}{3}U_{ZK}\left[\delta_6 \cos(\varphi_U - \frac{\pi}{3}) + \delta_2 \cos(\frac{2\pi}{3} - \varphi_U)\right] \tag{A.7}$$

$$u_{U,6} = |\underline{u}_{U,6}| = u_{U,2} = |\underline{u}_{U,2}| = \frac{2}{3}U_{ZK} \tag{A.8}$$

(the indices denote the converter switching state by the decimal equivalent of the converter switching status vector interpreted as binary number). There one basically has to distinguish between the microscopic or local time behavior (t_μ; $t_\mu \in [0, T_P/2]$) of the quantities within a pulse (half) period T_P and the macroscopic (or global) time behavior (τ or angle φ_U – identification of the pulse interval position within the fundamental period) of the quantities which are based on time averaging over a pulse period.

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Appendix B

For the quality functional we have

$$I = \Delta I^2_{N,RST,rms} = \frac{1}{T_N}\int_{T_N} \Delta i^2_{N,RST,rms}(\tau)\,d\tau \rightarrow Min \qquad (B.1)$$

the integral (related to the fundamental period) of the square of the deviation

$$\Delta \underline{i}_N = \underline{i}_N - \underline{i}^*_N \qquad (B.2)$$

between the current actual value and the reference value

$$\underline{i}^*_N(\tau) = \hat{\underline{I}}^*_N \exp j\left(\omega_N \tau + \varphi_{\underline{u}^*_U,\underline{i}^*_N}\right) . \qquad (B.3)$$

(This is equivalent to using the sum of the squares of the rms values of the current harmonics as quality functional.) For its calculation one can choose a simple equivalent circuit of the supplied system (considering the high pulse rate assumed) according to

$$L\,\frac{d\underline{i}_N}{dt} = (\underline{u}_U - \underline{u}_N) . \qquad (B.4)$$

For the space vector of $\Delta \underline{i}_N$ there follows (see Fig.3, Fig.(A.1) and Fig.(A.2))

$$\frac{d\Delta \underline{i}_N}{dt_\mu} = \frac{1}{L}\left[\underline{u}_U - \underline{u}^*_U(\tau)\right] \qquad (B.5)$$

$$\begin{aligned}
\Delta \underline{i}_{N,t_{\mu,1}}(\tau) &= \delta_7 \left[-\underline{u}^*_U(\tau)\right] \frac{T_P}{2L} \\
\Delta \underline{i}_{N,t_{\mu,2}}(\tau) &= \Delta \underline{i}_{N,t_{\mu,1}}(\tau) + \delta_6 \left[\underline{u}_{U,6} - \underline{u}^*_U(\tau)\right] \frac{T_P}{2L} \qquad (B.6) \\
\Delta \underline{i}_{N,t_{\mu,3}}(\tau) &= \Delta \underline{i}_{N,t_{\mu,2}}(\tau) + \delta_2 \left[\underline{u}_{U,2} - \underline{u}^*_U(\tau)\right] \frac{T_P}{2L}
\end{aligned}$$

or

$$\begin{aligned}
\Delta \underline{i}_{N,t_{\mu,2}}(\tau) &= \delta_6 \underline{u}_{U,6} \frac{T_P}{2L} - (\delta_7 + \delta_6)\, \underline{u}^*_U(\tau) \frac{T_P}{2L} \\
\Delta \underline{i}_{N,t_{\mu,3}}(\tau) &= \delta_0 \underline{u}^*_U(\tau) \frac{T_P}{2L} , \qquad (B.7)
\end{aligned}$$

respectively, with

$$L\,\frac{d\underline{i}^*_N}{dt} = (\underline{u}^*_U - \underline{u}_N) \qquad (B.8)$$

and

$$\begin{aligned}
\underline{u}^*_U(\tau) &= \underline{u}_N(\tau) + j\omega L\, \underline{i}^*_N(\tau) \\
\underline{i}^*_N(\tau + t_\mu) &= \underline{i}^*_N(\tau) + j\omega t_\mu\, \underline{i}^*_N(\tau) \qquad (B.9)
\end{aligned}$$

(approximation of the circular trajectory of the reference-current space vector by the local tangent). Then we have

$$\Delta \underline{i}_{N,\frac{1}{2}T_P}(\tau) = \underline{i}_{N,t_{\mu,3}}(\tau) - \delta_0 \underline{u}^*_U(\tau) \frac{T_P}{2L} = 0 \qquad (B.10)$$

with

$$\underline{u}^*_U(\tau) = \delta_6(\tau)\, \underline{u}_{U,6} + \delta_2(\tau)\, \underline{u}_{U,2} \qquad (B.11)$$

or

$$\underline{i}^*_N\left(\tau + \frac{T_P}{2}\right) = \left(1 + j\omega_N \frac{T_P}{2}\right) \underline{i}^*_N(\tau) , \qquad (B.12)$$

respectively. As is shown by a simple consideration, one can formulate the sum of the squares of the phase currents via the α,β-coordinates of the related space vector

$$\Delta i^2_{N,R} + \Delta i^2_{N,S} + \Delta i^2_{N,T} = \frac{3}{2}[\Delta i^2_{N,\alpha} + \Delta i^2_{N,\beta}] = \frac{3}{2}|\Delta \underline{i}_N|^2 . \qquad (B.13)$$

With this, Eq.(B.6) and

$$\begin{aligned}
&\frac{1}{(t_{\mu,i+1} - t_{\mu,i})} \int_{t_{\mu,i}}^{t_{\mu,i+1}} \left[\Delta i^2_{N,\alpha}(t_\mu) + \Delta i^2_{N,\beta}(t_\mu)\right] dt_\mu = \\
&\frac{1}{3}\left[\left(\Delta i^2_{N,i,\alpha} + \Delta i_{N,i,\alpha}\Delta i_{N,i+1,\alpha} + \Delta i^2_{N,i+1,\alpha}\right) + \right. \\
&\left. + \left(\Delta i^2_{N,i,\beta} + \Delta i_{N,i,\beta}\Delta i_{N,i+1,\beta} + \Delta i^2_{N,i+1,\beta}\right)\right] \qquad (B.14)
\end{aligned}$$

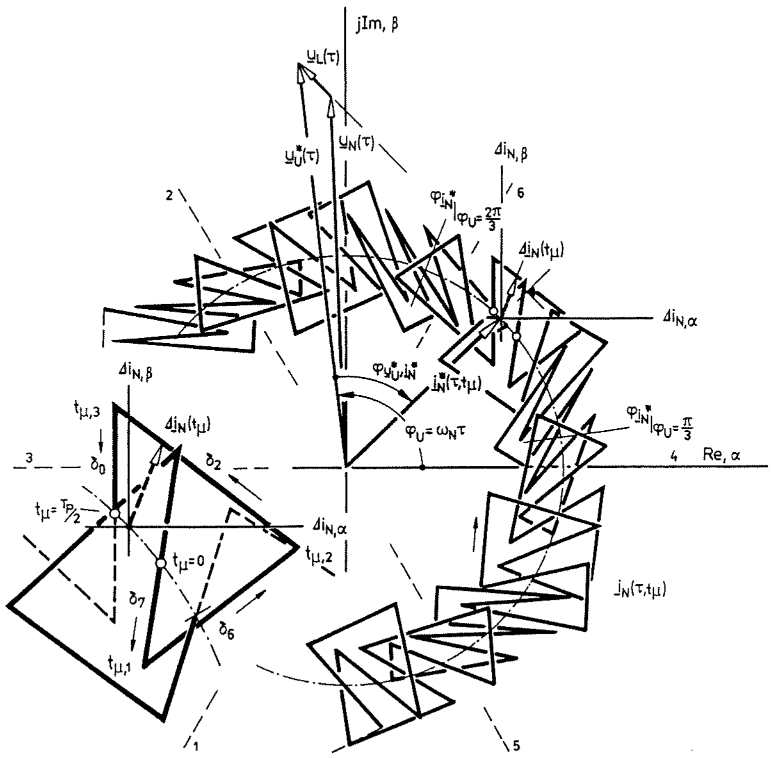

Fig.(A.1): Illustration of the definition of the current deviation according to Eq.(16) ($\Delta \underline{i}_N$ ... space vector of the converter output current).

the local rms value (related to a pulse (half) interval) of the current ripple can be expressed according to

$$\Delta i^2_{N,RST,rms}(\tau) =$$
$$\frac{2}{T_P}\int_{t_\mu=0}^{t_\mu=\frac{1}{2}T_P}\left[\Delta i^2_{N,R}(t_\mu)+\Delta i^2_{N,S}(t_\mu)+\Delta i^2_{N,T}(t_\mu)\right]\,dt_\mu =$$
$$\begin{aligned} = \tfrac{1}{2}\Big\{ &\delta_7 \;\left[\Delta i^2_{N,t_{\mu,1},\alpha} + \Delta i^2_{N,t_{\mu,1},\beta}\right] \\ +&\delta_6 \;\Big[\Delta i^2_{N,t_{\mu,1},\alpha} + \Delta i^2_{N,t_{\mu,1},\beta} + \Delta i^2_{N,t_{\mu,2},\alpha} + \Delta i^2_{N,t_{\mu,2},\beta} + \\ &\qquad + \Delta i_{N,t_{\mu,1},\alpha}\Delta i_{N,t_{\mu,2},\alpha} + \Delta i_{N,t_{\mu,1},\beta}\Delta i_{N,t_{\mu,2},\beta}\Big] + \\ +&\delta_2 \;\Big[\Delta i^2_{N,t_{\mu,2},\alpha} + \Delta i^2_{N,t_{\mu,2},\beta} + \Delta i^2_{N,t_{\mu,3},\alpha} + \Delta i^2_{N,t_{\mu,3},\beta} + \\ &\qquad + \Delta i_{N,t_{\mu,2},\alpha}\Delta i_{N,t_{\mu,3},\alpha} + \Delta i_{N,t_{\mu,2},\beta}\Delta i_{N,t_{\mu,3},\beta}\Big] + \\ +&\delta_0 \;\left[\Delta i^2_{N,t_{\mu,3},\alpha} + \Delta i^2_{N,t_{\mu,3},\beta}\right]\Big\} \end{aligned} \tag{B.15}$$

with

$$\delta_0 = 1 - (\delta_7 + \delta_6 + \delta_2)\ . \tag{B.16}$$

For the calculation of the global rms value (being set equal to the quality functional I) related to the fundamental period we have

$$\begin{aligned} I &= \Delta I^2_{N,RST,rms} = \frac{1}{T_N}\int_{T_N}(\Delta i^2_{N,R} + \Delta i^2_{N,S} + \Delta i^2_{N,T})\,dt \\ &= \frac{1}{T_N}\sum_j \int_{\frac{1}{2}T_P(j)}(\Delta i^2_{N,R} + \Delta i^2_{N,S} + \Delta i^2_{N,T})\,dt_\mu\ . \end{aligned} \tag{B.17}$$

Now, in the sense of a simple averaging of the local harmonic loss contributions the position of the pulse interval shall be moved time-continuously through the fundamental period. Then with sufficiently good approximation for high pulse frequency, the summation due to the finite system switching frequency can be replaced by an integration. This allows to calculate a simple analytical expression for the harmonic power loss

$$\begin{aligned} I = \Delta I^2_{N,RST,rms} &= \\ &= \frac{1}{T_N}\int_{T_N}\Big\{\frac{2}{T_P}\int_{t_\mu=0}^{t_\mu=\frac{1}{2}T_P}(\Delta i^2_{N,R} + \Delta i^2_{N,S} + \Delta i^2_{N,T})\,dt_\mu\Big\}\,d\tau \\ &= \frac{1}{T_N}\int_{T_N}\Delta i^2_{N,RST,rms}(\tau)\,d\tau \rightarrow Min \end{aligned} \tag{B.18}$$

via the local harmonic rms value

$$\begin{aligned} \Delta i^2_{N,RST,rms}(\tau) &= \Delta i^2_{N,RST,rms,1}\{\delta_7(\tau),\delta_6(\tau),\delta_2(\tau)\} + \\ &+ \Delta i^2_{N,RST,rms,2}\{\delta_6(\tau),\delta_2(\tau)\} \end{aligned} \tag{B.19}$$

$$\frac{1}{\Delta i^2_n}\Delta i^2_{N,RST,rms,1} =$$
$$\frac{48}{9}\delta_7\Big\{\delta_2\delta_6(\delta_2-\delta_6) - 2\delta_{26}\left[1-(\delta_7+\delta_6+\delta_2)\right]\Big\} \tag{B.20}$$

$$\frac{1}{\Delta i^2_n}\Delta i^2_{N,RST,rms,2} =$$
$$\begin{aligned} \frac{16}{9}\Big\{&6\delta^2_{26} + 2\delta_{26} - 4\delta^4_2 - 4\delta^4_6 - 4\delta_2\delta_{26} - 4\delta_6\delta_{26} - \\ &- 8\delta_2\delta_6\delta_{26} + 2\delta_2\delta^2_6 - \delta^2_2\delta_6 + 3\delta^3_2\delta_6 - \delta^2_2\delta^2_6\Big\} \end{aligned} \tag{B.21}$$

with

$$\Delta i_n = \frac{U_{ZK}T_P}{8L} \tag{B.22}$$

and

$$\delta_{26} = \delta^2_2 + \delta_2\delta_6 + \delta^2_6\ . \tag{B.23}$$

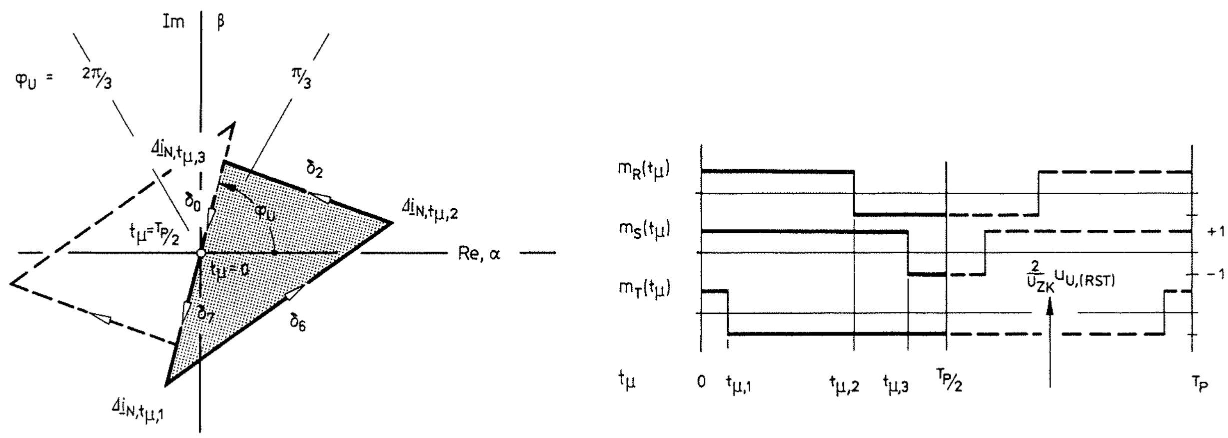

Fig.(A.2): left: Trajectory of the space vector $\Delta \underline{i}_N(t_\mu)$ within one pulse half period; right: associated microscopic time behavior of the phase modulation functions (pulse pattern).