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MEASUREMENT AND DETERMINATION OF THE INSTANTANEOUS POWER FLOW VALUES IN THREE-PHASE PWM CONVERTER SYSTEMS BASED ON THE SPACE VECTOR CALCULUS

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Summary

In analogy to the theory of electrical machines also the operational behavior of a DC link pulse width modulated (PWM) converter system can be described with minimum effort if a method corresponding to the space vector calculus is used. This method mentioned gives a power-invariant transformation of the phase variables into corresponding image variables. This topic seems important because PWM converters receive increasing attention and gain more and more importance in modern power electronics, especially in applications to AC motor drives.

As this paper will show, this method makes it possible to define an expression to be called "instantaneous reactive power" additionally to the instantaneous value of the resulting power flow. This assumes restriction to a three phase system (decoupling of the zero-sequence system). This "instantaneous reactive power" describes the currents which in each instant as a sum do not contribute to the power flow. Also, this can be interpreted as energy exchange between the phases of the AC current system.

In accordance to the space vector calculus this definition is valid independent of time behavior, of harmonic content and of the symmetry of the underlying current-voltage-system. This means that this definition is not restricted to a description based on the fundamental and it does not assume determination of an average value as power definitions of the classical AC current calculus do (e.g., real power, reactive power, distortional power,...). The orthogonal instantaneous power components derived therefore are ideally applicable to the physically illustrative description of the stationary and transient behaviour of power electronic systems. This will be shown for the example of a PWM converter system with a constant voltage DC link.

Concluding considerations are related to measurement and determination of power flow components with circuits of limited complexity and to the inclusion of this measurement and determination procedure into control concepts based on transformation of system values into space vectors.

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Introduction

As introduction into the problem area of this paper we want to review the power relations of a symmetric, purely sinusoidal three wire mains system before deriving general relations (valid for any current and voltage shape). There we shall have for the voltage and current system

$$\begin{aligned} u_{N,R} &= \hat{U}_N \cos\left[\omega_N t + \varphi\right] \\ u_{N,S} &= \hat{U}_N \cos\left[\omega_N \left(t - \tfrac{T}{3}\right) + \varphi\right] \\ u_{N,T} &= \hat{U}_N \cos\left[\omega_N \left(t + \tfrac{T}{3}\right) + \varphi\right] , \end{aligned} \tag{1}$$

$$\begin{aligned} i_{N,R} &= \hat{I}_N \cos \ \omega_N t \\ i_{N,S} &= \hat{I}_N \cos\left[\omega_N \left(t - \tfrac{T}{3}\right)\right] \\ i_{N,T} &= \hat{I}_N \cos\left[\omega_N \left(t + \tfrac{T}{3}\right)\right] . \end{aligned} \tag{2}$$

Then, for the instantaneous total power of the three phases there follows according to

$$p_N(t) = u_{N,R}\, i_{N,R} + u_{N,S}\, i_{N,S} + u_{N,T}\, i_{N,T} \tag{3}$$

the relation

$$\begin{aligned} p_N(t) &= \tfrac{P_N}{3}\left[1 + \cos 2\omega_N t\right] &&+ \tfrac{Q_N}{3} \sin 2\omega_N t \\ &+ \tfrac{P_N}{3}\left[1 + \cos 2\omega_N \left(t - \tfrac{T}{3}\right)\right] &&+ \tfrac{Q_N}{3} \sin 2\omega_N \left(t - \tfrac{T}{3}\right) \\ &+ \tfrac{P_N}{3}\left[1 + \cos 2\omega_N \left(t + \tfrac{T}{3}\right)\right] &&+ \tfrac{Q_N}{3} \sin 2\omega_N \left(t + \tfrac{T}{3}\right) \end{aligned} \tag{4}$$

with

$$P_N = \frac{3}{2}\hat{U}_N\, \hat{I}_N \cos\varphi \qquad\qquad Q_N = \frac{3}{2}\hat{U}_N\, \hat{I}_N \sin\varphi \ . \tag{5}$$

There a split of the power contributions of the phases into an always positive part (defining the real power P_N) and into a part being symmetrical to zero (defining the reactive power Q_N) has been performed. The superposition of the power flow of the different phases ("phase power flow components") leads to the constant instantaneous power (equal to the real power) characterizing a symmetrical three-phase mains and to power contributions which have the sum of zero in each point of time

$$\frac{Q_N}{3}\left[\sin 2\omega_N t + \sin 2\omega_N \left(t - \tfrac{T}{3}\right) + \sin 2\omega_N \left(t + \tfrac{T}{3}\right)\right] \ \equiv \ 0 \ . \tag{6}$$

These contributions are given by the electrical and magnetic energy storage elements. This immediately leads to the time constant energy content

$$W_{mag} \ = \ \frac{3}{4} L\, \hat{I}_N^2 \qquad\qquad W_{el} \ = \ \frac{3}{4} C\, \hat{U}_N^2 \tag{7}$$

of a symmetric electric or magnetic storage of energy. This means that there only takes place a "re-distribution" of the energy contributions among the phases. Furthermore, this immediately points out the possibility of modeling the behavior of a three-phase storage of energy by a power electronic circuit which has to guarantee the energy exchange between the phases as mentioned.

An intermediate storage of energy therefore is not necessary. Therefore, with theoretically ideal function of the converter circuit (infinitely high switching frequency), the "simulation" of an inductive or capacitive behavior without real energy storage devices is possible by switching devices only.

Due to this interesting aspect the idea of reactive power compensation by an ("active") power electronic unit, in general to be called static var compensator, is near at hand. However, there the real conditions in electrical energy supplying mains (unsymmetry, harmonics, ...) have to be considered by modification of the ideal conditions mentioned initially.

Definition of the Power Flow Components in a Three-Wire System by Application of Space Vector Calculus [1], [2], [3], [4], [5], [6], [7], [8], [9]

For describing the stationary and dynamic working behavior of a general three-wire system the space vector calculus known from the theory of electrical machines suggests itself. There the space vector transformation in general transforms three linearly dependent (phase) quantities into the linearly independent components (which are sufficient for a complete representation) of the space vector being defined in the complex plane (see Fig.1). For the definition of the space vector we have

$$\underline{u}_N = \frac{2}{3}\left[u_{N,R} + \underline{a}\, u_{N,S} + \underline{a}^2\, u_{N,T}\right] \qquad \underline{a} = \exp j\tfrac{2\pi}{3} \quad \underline{a}^2 = \exp j\tfrac{-2\pi}{3} \; . \tag{8}$$

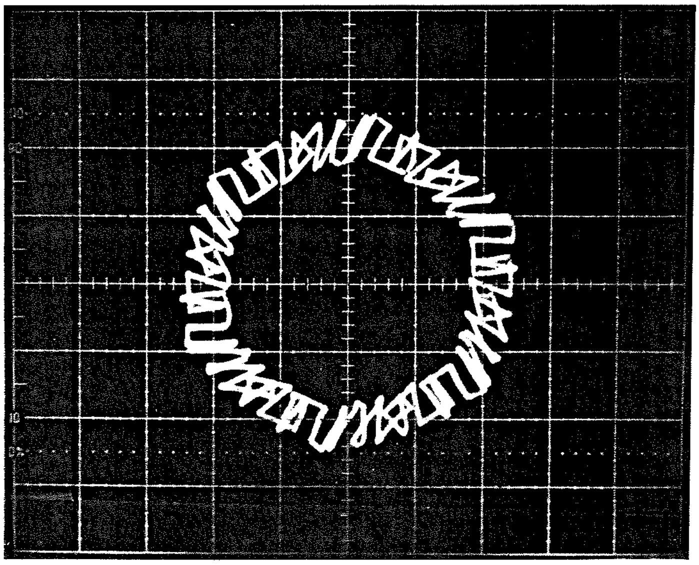

Fig.1. Trajectory of stator current space vector of an induction motor supplied by a voltage DC link PWM converter (asymmetric regular sampling, pulse number pz=21, horizontal: α-axis, vertical: β-axis)

A zero-sequence system being contained in general in the phase voltages (linear independence of the phase voltages due to the free choice of the reference potential) thereby is decoupled according to

$$u_{N,0} + \underline{a}\, u_{N,0} + \underline{a}^2\, u_{N,0} \;=\; 0 \; , \tag{9}$$

$$\frac{2}{3}\left(u_{N,R} + \underline{a}\, u_{N,S} + \underline{a}^2\, u_{N,T}\right) \;\equiv\; \frac{2}{3}\left(u'_{N,R} + \underline{a}\, u'_{N,S} + \underline{a}^2\, u'_{N,T}\right) \; . \tag{10}$$

Thereby, however, there is given no limitation of the representation of three-phase systems according to

$$p_N(t) \;=\; u'_{N,R}\, i'_{N,R} + u'_{N,S}\, i'_{N,S} + u'_{N,T}\, i'_{N,T} + 3u_{N,0}\, i_{N,0} \; , \tag{11}$$

$$u'_{N,RST} = u_{N,RST} - u_{N,0} \;, \tag{12}$$

$$u_{N,0} = \frac{1}{3}\left[u_{N,R} + u_{N,S} + u_{N,T}\right]$$

$$i_{N,0} = \frac{1}{3}\left[i_{N,R} + i_{N,S} + i_{N,T}\right] = 0 \;. \tag{13}$$

This is because, due to the first law of Kirchhoff, there does not exist a current zero-sequence system which could cause a power flow in connection with the voltage zero-sequence system.

For the calculation of the instantaneous power of the three-wire system we have to apply the relation

$$p_N(t) = \frac{3}{2}\,\Re\left\{\underline{u}_N\,\underline{i}_N^*\right\} \tag{14}$$

when space vector calculus is applied. The evaluation of Eq.(14) in phase quantities leads to

$$\begin{aligned} p_N(t) &= \Re\,\Bigg\{\Big[u_{N,R}\,i_{N,R} + u_{N,S}\,i_{N,S} + u_{N,T}\,i_{N,T}\Big] \\ &+ j\frac{1}{\sqrt{3}}\Big[u_{N,R}(i_{N,T} - i_{N,S}) + u_{N,S}(i_{N,R} - i_{N,T}) + u_{N,T}(i_{N,S} - i_{N,R})\Big]\Bigg\} \;. \end{aligned} \tag{15}$$

The resulting instantaneous power flow of the phases therefore is determined according to

$$p_N(t) = \Re\left\{\underline{s}_N(t)\right\} \tag{16}$$

via the real component of a general instantaneous power space vector

$$\underline{s}_N(t) = p_N(t) + j\,q_N(t) = \left\{\underline{u}_N\,\underline{i}_N^*\right\} \;. \tag{17}$$

Because the current and voltage space vectors completely describe the current/voltage system in each point of time, for the imaginary component of $\underline{s}_N(t)$

$$\begin{aligned} q_N(t) &= \frac{1}{\sqrt{3}}\Big[u_{N,R}(i_{N,T} - i_{N,S}) + u_{N,S}(i_{N,R} - i_{N,T}) + u_{N,T}(i_{N,S} - i_{N,R})\Big] \\ &= \frac{1}{\sqrt{3}}\Big[i_{N,R}(u_{N,S} - u_{N,T}) + i_{N,S}(u_{N,T} - u_{N,R}) + i_{N,T}(u_{N,R} - u_{N,S})\Big] \;, \end{aligned} \tag{18}$$

the assumption suggests itself that thereby power flows between phases are characterized which do not contribute as a sum to the resulting energy flow. This can also be stated because (according to Eq.(18)) the phase and line-to-line quantities connected together have to be gained via projection of the space vectors to axes lying perpendicular with respect to each other. For a clearer representation of the physical conditions a description of the power quantities in α,β-coordinates (see Fig.2)

$$\underline{u}_N^{\alpha,\beta} = u_{N,\alpha} + j\,u_{N,\beta} \qquad \underline{i}_N^{\alpha,\beta} = i_{N,\alpha} + j\,i_{N,\beta} \tag{19}$$

$$p_N(t) = \frac{3}{2}(u_{N,\alpha}\,i_{N,\alpha} + u_{N,\beta}\,i_{N,\beta}) \tag{20}$$

$$q_N(t) = \frac{3}{2}(u_{N,\beta}\,i_{N,\alpha} - u_{N,\alpha}\,i_{N,\beta}) \tag{21}$$

or in r,φ-coordinates

$$\underline{u}_N^{r,\varphi} = |\underline{u}_N|\exp j\varphi_{\underline{u}_N} \qquad \underline{i}_N^{r,\varphi} = |\underline{i}_N|\exp j\varphi_{\underline{i}_N} \tag{22}$$

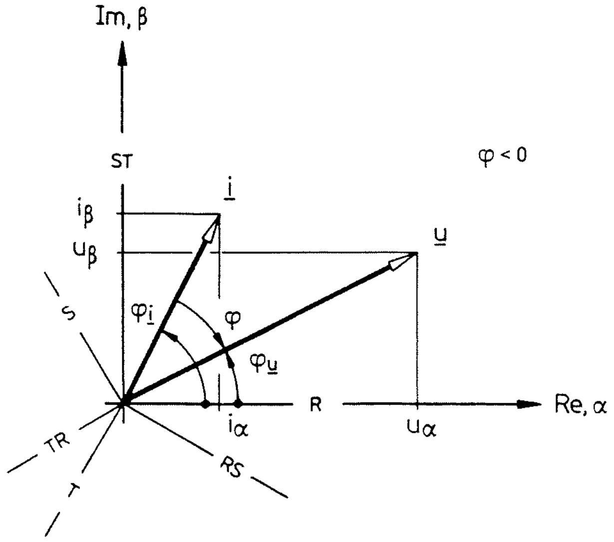

Fig.2. Representation of the current and voltage space vector in α,β-coordinates, definition of the direction of positive and negative angles

$$\varphi(t) \quad = \quad \varphi_{\underline{u}_N} - \varphi_{\underline{i}_N} \ , \tag{23}$$

$$p_N(t) = \frac{3}{2}|\underline{u}_N|\,|\underline{i}_N|\,\cos\varphi \qquad\qquad q_N(t) = \frac{3}{2}|\underline{u}_N|\,|\underline{i}_N|\,\sin\varphi \ , \tag{24}$$

respectively, appears favorable. Considering Eq.(18), the transfer of a defined instantaneous power $p_N(t)$ for required minimum current space vector magnitude is only possible for parallel current and voltage space vectors. This is equivalent to a symmetric resistive load for the three-line system (see Fig.4). According to Eqs.(21) or (24), respectively, then there is $q_N(t) = 0$ in each point of time. Due to

$$i_{N,R}^2 + i_{N,S}^2 + i_{N,T}^2 \quad = \quad \frac{3}{2}|\underline{i}_N|^2 \tag{25}$$

there exists a general proportionality between the transmission losses and the current space vector magnitude. Therefore, the energy transmission described previously is power loss optimal for the minimum current space vector magnitude. Therefore, there immediately is found a physically clear explanation of the imaginary component of the power space vector. For

$$q_N(t) \ \equiv \ 0 \qquad\qquad i_{N,q}(t) \ = \ |\underline{i}_N|\,\sin\varphi \ \equiv \ 0 \tag{26}$$

in each point of time the power losses connected with the transmission of $p_N(t)$ are minimized. Therefore, the phase power flows are used in an optimal manner for the energy conversion leading to an ideal use of the supply lines.

Assuming an arbitrary load current distribution (described by a space vector $\underline{i}_L$) we therefore can obtain a redistribution of the phase currents using a "compensator" such that the mains current i_N determined according to

$$\underline{i}_N \quad = \quad \underline{i}_L + \underline{i}_C \tag{27}$$

guarantees an energy transmission with minimal power loss. The compensator thereby only has to supply the part $q_L(t)$ and therefore no resulting instantaneous power; therefore the compensator can be realized by a power electronic circuit which does not contain a physical energy storage element (see Figs.3, 4). This assumes ideal function (as mentioned, infinitely high switching frequency). A compensation (smoothing) of power fluctuations on the load side thereby in principle is not possible, however. It has to be mentioned, however, that the "compensation strategy" given by Eq.(26) only represents one of different possibilities. E.g., another aim of the compensation is

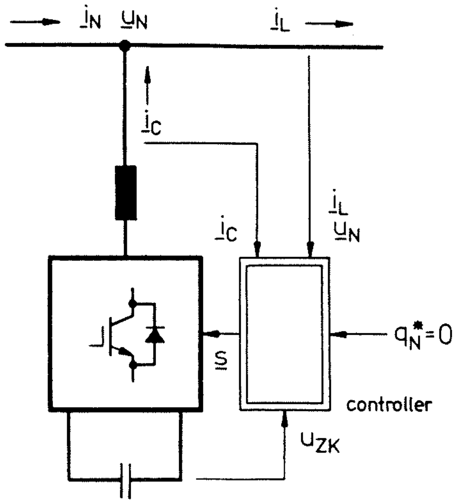

Fig.3. Basic circuit of a power electronic compensator (voltage DC link PWM converter)

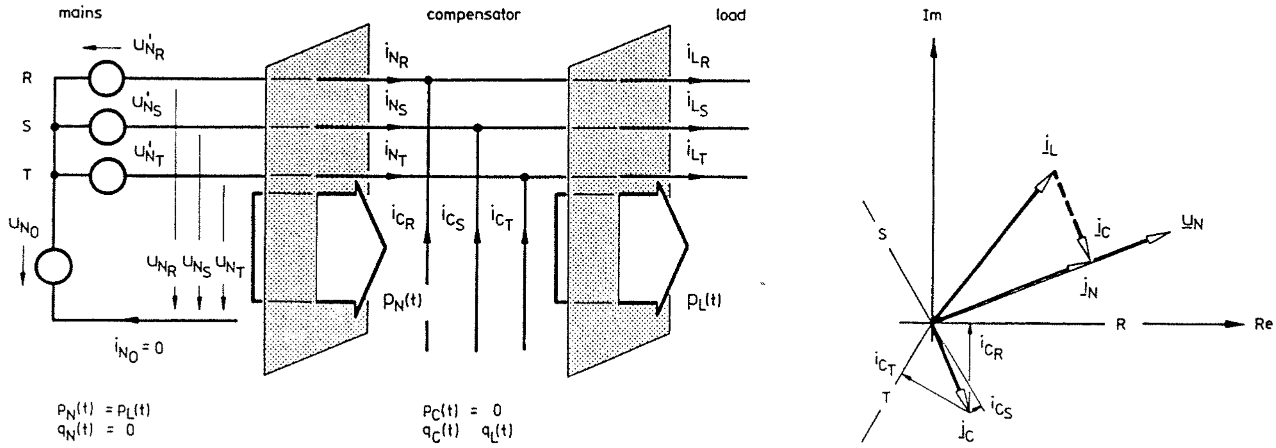

Fig.4. Principle of the compensation represented in phase and space vector quantities

thinkable, such as coupling the mains current system to the positive-sequence system of the mains voltage. Thereby, however, (as a more detailed analysis shows) only the mean value of the power delivered by the compensator becomes zero and not the instantaneous power of the compensator. This would require an energy storage device. According to the considerations made before, in the following the quantity $p_N(t)$ is called *instantaneous reactive power*. The terms *instantaneous real power* and *instantaneous reactive power* are chosen in that way because they turn into the well known terms *real power* and *reactive power* used in basic electrical engineering (where only linear and quadratic mean values are considered) for the symmetric stationary case (purely sinusoidal quantities) considered initially.

For illustrating the physical meaning of the appearance of an instantaneous reactive power we want to briefly consider here the power loss optimal energy transmission for a single-phase mains. For this the quotient $\lambda = P/S$ (power factor) defined according to

$$S = \sqrt{\frac{1}{T}\int_0^T u^2\,dt}\;\sqrt{\frac{1}{T}\int_0^T i^2\,dt} \quad = \quad U_{rms}\,I_{rms} \qquad (28)$$

(where S is the apparent power describing the load condition and

$$P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T u\,i\,dt \tag{29}$$

the transmitted power averaged in a time interval T, i.e. the real power) is only 1 if current and voltage are connected to each other by a time constant factor

$$\frac{u(t)}{i(t)} = R \tag{30}$$

("maximum correlation"). This is immediately clear by considering the CAUCHY-SCHWARZ inequality

$$\left[\int_a^b f(x)\,g(x)\,dx\right]^2 \leq \left[\int_a^b f^2(x)\,dx\right]\left[\int_a^b g^2(x)\,dx\right] \tag{31}$$

which turns for validity of

$$\frac{f(x)}{g(x)} = k \tag{32}$$

into

$$\int_a^b f(x)\,g(x)\,dx = \sqrt{\int_a^b f^2(x)\,dx}\,\sqrt{\int_a^b g^2(x)\,dx}\ . \tag{33}$$

The "optimal" load, therefore, in single-phase systems only can be determined "a posteriori" or, for an integration interval of constant length moving with time, "from the history". If only the instantaneous value of current and voltage are known, only magnitude and direction of the instantaneous power flow can be described; a separation of a "reactive part" naturally is impossible. Furthermore, one has to point out that a load resistance being not time constant already leads to the appearance of reactive power (which is a mean value!). Reactive power therefore is not always to be connected to the presence of electrical or magnetic storage devices or to a reversal of the power flow direction (ideal switch as ideal reactive element!).

If the single-phase system is extended to a three-phase system we also have to consider (besides the resulting power flow in all phases) the power exchange taking place between the phases; this exchange does not contribute to the power transmission. Under no circumstances one can consider the complete three-phase system as a combination of independent single-phase systems. As represented via the complex power (Eq.(17)) one can therefore (contrary to single-phase systems) define an instantaneous reactive power besides the instantaneous (real) power. The compensation of this instantaneous reactive power is only given for topographically equal load in all three phases (e.g., Y-connection of equal resistances). A time constant "optimal" load is not assumed in this connection.

If the system power flows are not considered in a special point of time but within a time interval a further question arises: the definition of an "optimal" (time constant) load for the power transmitted within this time interval. For this the definitions of basic electrical engineering are to be used, e.g. the apparent power definition according to

$$S_{m=3} = \sqrt{\sum_{i=R,S,T}\frac{1}{T}\int_0^T u_i^2\,dt}\,\sqrt{\sum_{i=R,S,T}\frac{1}{T}\int_0^T i_i^2\,dt}\ . \tag{34}$$

For the sake of brevity we do not want to go into further details of the associated problems. Instead, we always want to characterize the power flow components based on the knowledge of the instantaneous values of current and voltage, or we want to obtain the power transmission being optimal (i.e., with minimum loss) in any particular point of time.

Measurement of the Instantaneous Power Flow Components and Generating the Reference Values of the Inverter Currents [1]

After discussing the physical meaning of the real and imaginary components of the instantaneous power space vector we now want to discuss their measurement in more detail. This is of importance especially for inclusion of the power components into the control concept of a power electronic system (e.g., PWM rectifier, PWM inverter, static var compensator, ...) based on transformation of the system quantities into space vectors. The determination of $p_N(t)$ and $q_N(t)$ has to be performed based on Eqs.(20) and (21) (see Fig.5). There one has to apply the following equations

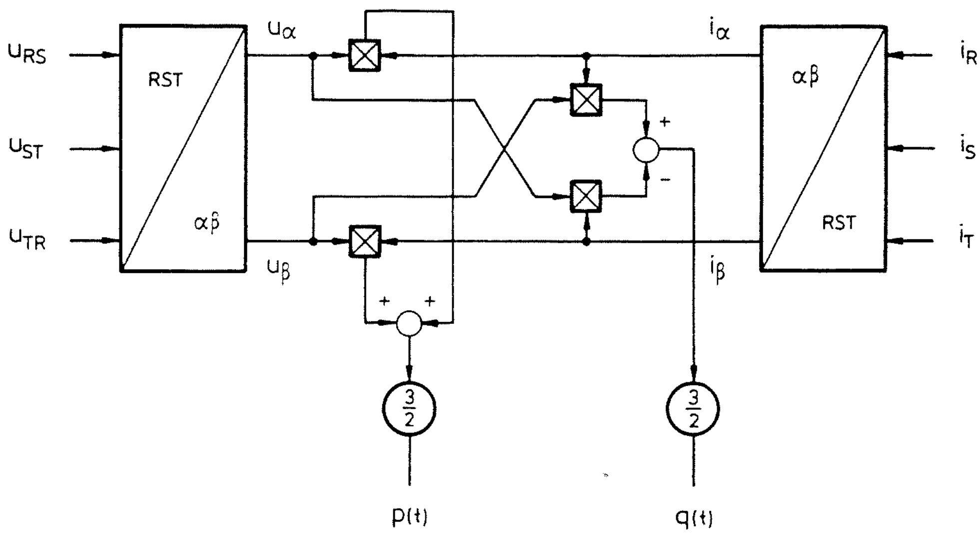

Fig.5. Measurement of the instantaneous real and reactive power flow based on the space vector calculus

$$u_\alpha = \frac{1}{3}(u_{RS} + u_{RT}) = u_R - \frac{1}{3}(u_R + u_S + u_T) \tag{35}$$

$$u_\beta = \frac{1}{\sqrt{3}} u_{ST} \ , \tag{36}$$

(which are shown for the voltages as example) for transformation of the phase quantities into α, β-coordinates. Based on the difference of the actual values and the reference values of the power components furthermore approriate control signals for the inverter power devices (magnitude and phase of the converter voltage space vector (averaged over a pulse period)) have to be derived via controllers for the instantaneous real power and for the instantaneous reactive power. Besides the "detour" via α, β-coordinates also the formulation of $q_N(t)$ and $p_N(t)$ is of course possible via the definition of the phase quantities (Eqs.(3), (18)). There it is sensible to take the opportunity of simplifications according to the linear dependency of the line-to-line voltages and the phase currents.

If the converter control is achieved via an inner loop current control, the phase current reference values on the other hand have to be derived based on the reference values of $p(t)$ and $q(t)$ according to (see Fig.6)

$$i_\alpha = \frac{2}{3}\frac{p\, u_\alpha}{(u_\alpha^2 + u_\beta^2)} + \frac{2}{3}\frac{q\, u_\beta}{(u_\alpha^2 + u_\beta^2)} = i_{\alpha,p} + i_{\alpha,q} \tag{37}$$

$$i_\beta = \frac{2}{3}\frac{p\,u_\beta}{(u_\alpha^2+u_\beta^2)} - \frac{2}{3}\frac{q\,u_\alpha}{(u_\alpha^2+u_\beta^2)} = i_{\beta,p} + i_{\beta,q}\ . \tag{38}$$

There for the inverse transformation of the α,β-coordinates into phase quantities free of a zero-

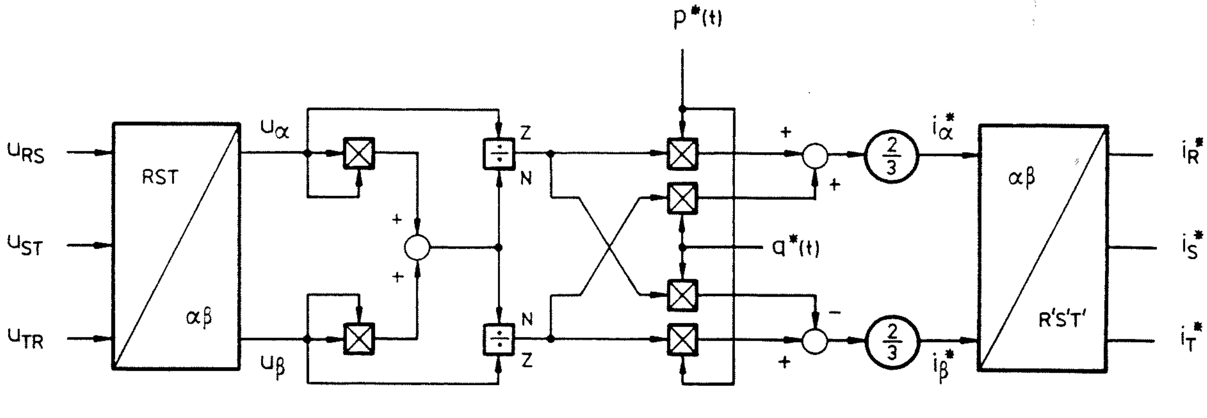

Fig.6. Transformation of the instantaneous real and reactive power reference values into phase current reference values

sequence system we have

$$u'_R = u_\alpha \qquad u'_S = -\frac{1}{2}u_\alpha + \frac{\sqrt{3}}{2}u_\beta \qquad u'_T = -\frac{1}{2}u_\alpha - \frac{\sqrt{3}}{2}u_\beta \tag{39}$$

where again the representation for the voltage is used as example. For the components giving the instantaneous real and reactive power of the α component (which directly corresponds to the phase current i_R) there follows

$$i_{\alpha,p} = \frac{3p\,u'_R}{(u_{RS}^2 + u_{ST}^2 + u_{TR}^2)} \tag{40}$$

$$i_{\alpha,q} = \frac{u_{ST}(i_R\,u_{ST} + i_S\,u_{TR} + i_T\,u_{RS})}{(u_{RS}^2 + u_{ST}^2 + u_{TR}^2)}\ . \tag{41}$$

Optimization of the Energy Transmission in a Three-Wire System [10],[11]

The determination of phase current reference values treated in the previous section is now considered for a compensator (e.g., realized by a voltage or current DC-link PWM converter) in parallel to the load (being possibly, e.g., also a complete section of the mains). There, the compensator has to supply the instantaneous reactive power without influencing the resulting instantaneous real power flow. The reference value of the compensator phase current (phase R) then is determined by Eq.(41). There one has to insert the instantaneous values of the load currents for the phase currents. With this there follows a minimum instantaneous value of the transmission losses between the supplying mains and the load. This simply can be checked also by an optimizing calculation in phase quantities. This calculation has to be performed under the additional conditions

$$u_{N,R}\,i_{C,R} + u_{N,S}\,i_{C,S} + u_{N,T}\,i_{C,T} = p_C \tag{42}$$

$$i_{C,R} + i_{C,S} + i_{C,T} = 0 \tag{43}$$

for the optimization criterion

$$(i_{L,R} - i_{C,R})^2 + (i_{L,S} - i_{C,S})^2 + (i_{L,T} - i_{C,T})^2 = J \rightarrow min \ . \tag{44}$$

By the additional condition Eq.(42) the power consumption of the compensator is defined. (Ideally, this would be zero; in reality, however, the compensator losses have to be covered. This has to be performed by adjusting of p_N by the DC link voltage (or current) controller.) The additional conditions have to be introduced because otherwise the compensator would supply the total instantaneous power corresponding to a trivial solution of the optimization problem. For the optimal mains currents we have

$$i_{N,RST} \quad = \quad i_{L,RST} + i_{C,RST} \ . \tag{45}$$

The compensator phase currents are given by

$$i_{C,R} \quad = \quad \frac{3p_C\left[u_{N,R} - \frac{1}{3}(u_{N,R} + u_{N,S} + u_{N,T})\right]}{(u_{N,RS}^2 + u_{N,ST}^2 + u_{N,TR}^2)} + \frac{u_{ST}(i_R\, u_{N,ST} + i_S\, u_{N,TR} + i_T\, u_{N,RS})}{(u_{N,RS}^2 + u_{N,ST}^2 + u_{N,TR}^2)} \ , \tag{46}$$

represented for phase R, the other phases can be treated by cyclic interchange. As a comparison with Eqs.(40) or (41) shows, the results gained by using the optimization calculation based on phase quantities are identical to the results gained using space vector calculus. For the relation between compensator and load currents we have

$$i_{C,R} \quad = \quad i_{C,R,p} + i_{C,R,q} \qquad\qquad i_{C,RST,q} \quad = \quad i_{L,RST,q} \tag{47}$$

according to the remarks made previously for the supply of the instantaneous load reactive power by the compensator.

If the compensation ("re-distribution") is not treated for a load current system, but for the optimal (minimum loss) reference value determination of a mains current shape for given mains voltage system and a given instantaneous power p_N to be transmitted, then, according to

$$u_{N,R}\, i_{N,R} + u_{N,S}\, i_{N,S} + u_{N,T}\, i_{N,T} \quad = \quad p_N \tag{48}$$

$$i_{N,R} + i_{N,S} + i_{N,T} \quad = \quad 0 \tag{49}$$

$$i_{N,R}^2 + i_{N,S}^2 + i_{N,T}^2 \quad = \quad J \rightarrow min \ , \tag{50}$$

we receive

$$i_{N,R} = \frac{3p_N\left[u_{N,R} - \frac{1}{3}(u_{N,R} + u_{N,S} + u_{N,T})\right]}{(u_{N,RS}^2 + u_{N,ST}^2 + u_{N,TR}^2)} \tag{51}$$

(currents in phases S and T follow by cyclic interchange); giving the optimal mains current shape mentioned is equivalent to a direct calculation of the mains current system resulting for compensation of the load current system.

The optimization can be illustrated graphically in form of an elliptic paraboloid (defined by Eqs.(43) and (44)) with a plane given by Eqs.(42) and (43) (see Fig.7). The optimum is identical with the vertex of the parabola resulting as intersecting curve of both surfaces. Equation (51) also already has been derived in the course of the application of the space vector calculus (Eq.(40)). This is also shown by the possible replacement of the optimization criterion (eq.(44)) by the relation

$$u_{N,ST}\, i_{N,R} + u_{N,TR}\, i_{N,S} + u_{N,RS}\, i_{N,T} \quad = \quad q_N \quad = \quad 0 \ , \tag{52}$$

For the sake of completeness we finally want to consider here the optimization of the energy transmission of a four-wire system. This in general cannot be described completely by the space

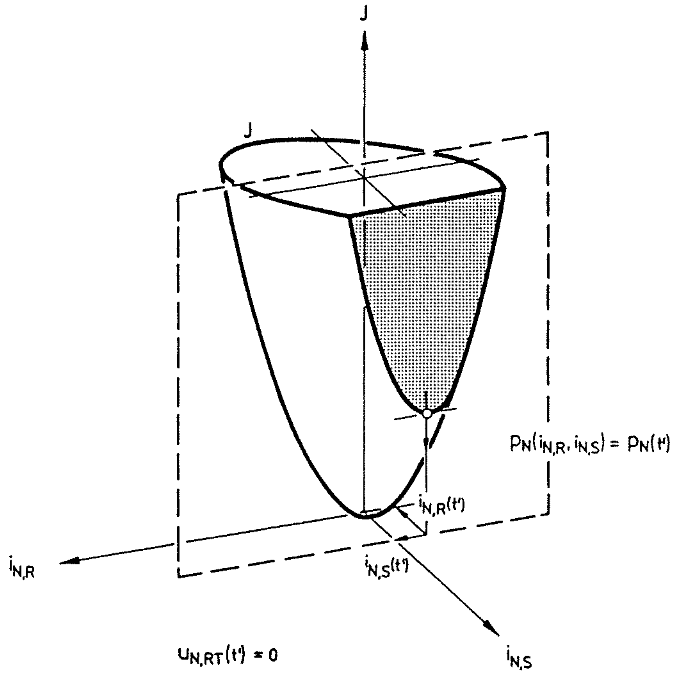

Fig.7. Graphic illustration of the minimization of the transmission losses of a three-phase system

vectors defined here. The mathematical relations then have to be formulated in phase quantities according to

$$u_{N,R}\, i_{C,R} + u_{N,S}\, i_{C,S} + u_{N,T}\, i_{C,T} = 0 \tag{53}$$

$$(i_{L,R} - i_{C,R})^2 + (i_{L,S} - i_{C,S})^2 + (i_{L,T} - i_{C,T})^2 + [(i_{L,R} + i_{L,S} + i_{L,T}) - (i_{C,R} + i_{C,S} + i_{C,T})]^2 = J \rightarrow min\ . \tag{54}$$

The compensator currents follow as

$$i_{C,R} = i_{L,R} - \frac{4p_L\left[u_{N,R} - \frac{1}{4}(u_{N,R} + u_{N,S} + u_{N,T})\right]}{\left[u^2_{N,RS} + u^2_{N,ST} + u^2_{N,TR} + u^2_{N,R} + u^2_{N,S} + u^2_{N,T}\right]}\ . \tag{55}$$

Equation (55) shows directly that the compensator has to accept the difference between the phase load current and the current giving the instantaneous power. There the compensator is of more general structure, i.e., it is not simply realizable by a forced commutated three-phase bridge circuit. The optimum mains (or load) current shape follows immediately via $i_{C,RST} = 0$. We now want to consider purely theoretically the case of not weighting the neutral current according to the losses introduced by it; then there follows

$$i_{C,R} = i_{L,R} - \frac{p_L\, u_{N,R}}{\left[u^2_{N,R} + u^2_{N,S} + u^2_{N,T}\right]}\ . \tag{56}$$

There, (for a closer analysis) the then given operating conditions would have to be compared to these conditions given for a compensation according to Eq.(55). This shall not be done here for the sake of brevity.

Analysis of the Voltage-DC-Link PWM Converter System [12],[13],[14]

In order to satisfy the compensating condition given by Eq.(26) in each point of time we have to assume the free reference value specification of a compensating current space vector. Impressing this space vector can be achieved, e.g., by using a forced commutated voltage or current DC link PWM converter system (see Fig.8). Its function will be treated in the following concerning the power flows, especially relating to the generation of the instantaneous reactive power.

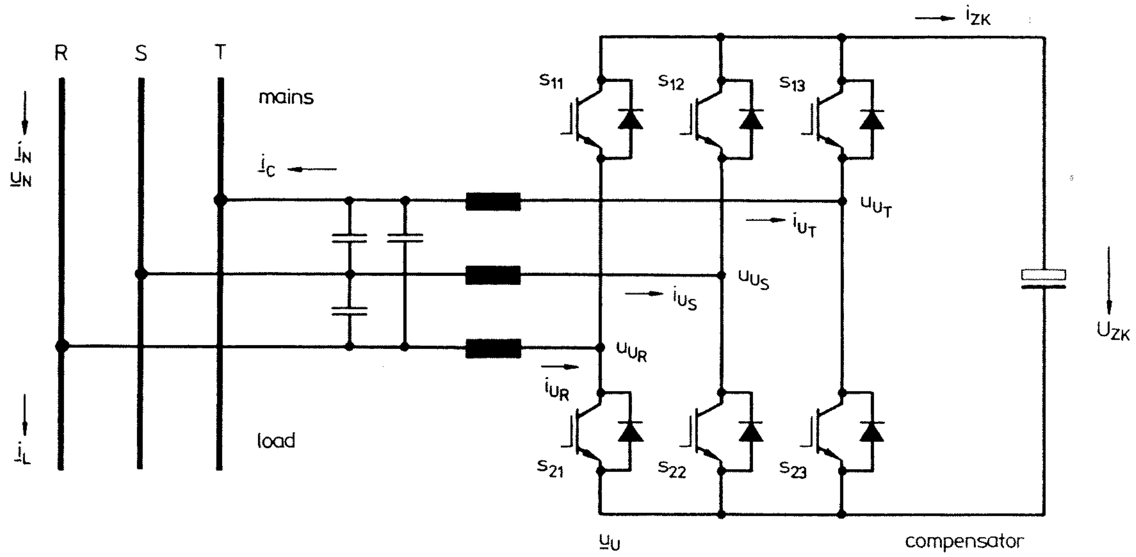

Fig.8. Realization of a compensator by a voltage DC link PWM converter system

For a voltage DC link PWM converter system a converter current (compensator current) space vector can be implemented practically without delay via the equilibrium of mains and converter voltages across the inductances in series. The system switching status can be completely described in each point of time by the switching status matrix

$$\mathbf{S} \;=\; \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \end{bmatrix} . \tag{57}$$

There the elements s_{ij} can assume the values 0 or 1 because they represent binary switching functions. Because a bridge leg can be replaced concerning its function by a two-pole switch between the positive and the negative DC link bus, we have as limiting condition

$$s_{11} + s_{21} = 1 \qquad s_{12} + s_{22} = 1 \qquad s_{13} + s_{23} = 1 \; . \tag{58}$$

Besides the description by phase switching functions we also can use the characterization by a switching status space vector defined by

$$\underline{s} \;=\; \frac{2}{3}\left[s_{11} + \underline{a}\, s_{12} + \underline{a}^2\, s_{13}\right] \qquad\qquad |\underline{s}| = \frac{2}{3} \tag{59}$$

($s_R = s_{11}$, $s_S = s_{12}$, $s_T = s_{13}$). Then we have

$$\underline{u}_U \;=\; \underline{s}\, U_{ZK} \;=\; \frac{2}{3}\left[\frac{u_{U,R}}{u_{ZK}} + \underline{a}\,\frac{u_{U,S}}{u_{ZK}} + \underline{a}^2\,\frac{u_{U,T}}{u_{ZK}}\right] . \tag{60}$$

The real power supplied to the DC link (identically zero for the ideal compensator) can be calculated by the power balance

$$p_U(t) \;=\; U_{ZK}\, i_{ZK} \;=\; \frac{3}{2}\Re\left\{\underline{u}_U\, \underline{i}^*\right\} . \tag{61}$$

From this the DC link current follows as

$$i_{ZK} \;=\; \frac{3}{2}\,\Re\{\underline{s}\,\underline{i}_U^*\} \;=\; s_R\, i_{U,R} + s_S\, i_{U,S} + s_T\, i_{U,T} \;=\; \frac{p_U}{U_{ZK}}\,. \tag{62}$$

According to Eqs.(60) or (62) one can formulate mathematically simply the description of the "transformation" of AC side into DC side quantities (or vice versa) by the switching status space vector. The realization in a practical circuit of the DC link current according to Eq.(62) can also be achieved (based on the phase currents) simply and directly via appropriately controlled analog switches (see Fig.9). Thereby analog multipliers can be avoided. The inclusion of the output signal

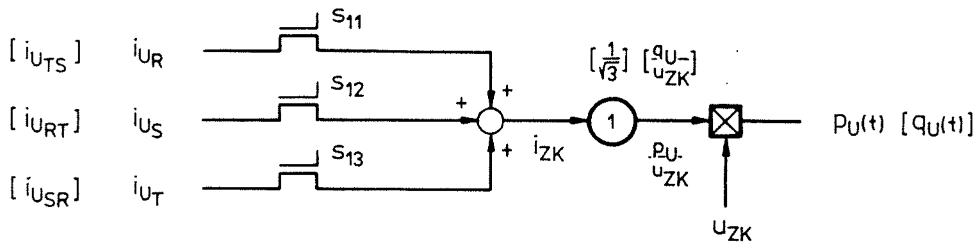

Fig.9. Determination of the instantaneous real and reactive power flow components of a voltage DC link PWM converter system (for forming the instantaneous reactive power the quantities given in parantheses are valid)

of the circuit described into the control of the instantaneous power flow (which can be realized by the converter system) suggests itself.

The circuit given before achieves the realization of the DC link current (of the instantaneous power) basically by direct representation of the function of the converter bridge half. The converter function therefore is completely described when also Eqs.(58) are considered. Based on this the derivation of the instantaneous reactive power appearing at the terminals

$$q_U(t) \;=\; \frac{3}{2}\,\Im\{\underline{u}_U\,\underline{i}^*\} \tag{63}$$

is possible by a circuit of equal structure. Then, however, according to

$$\frac{1}{U_{ZK}}\,q_U(t) \;=\; \frac{3}{2}\,\Im\,\{\underline{s}\,\underline{i}_U^*\} \;=\; \frac{1}{\sqrt{3}}\Big[s_R\,(i_{U,T} - i_{U,S}) + s_S\,(i_{U,R} - i_{U,T}) + s_T\,(i_{U,S} - i_{U,R})\Big] \tag{64}$$

the phase currents have to be replaced by "line-to-line currents" and a correction of the output signal amplitudes has to be performed (see Figs.9–12). Table 1 shows a summary of the instantaneous power expressions resulting for different switching states.

Dependent on the converter switching status there occurs always a short circuit between at least two phases of the three-wire system. The difference of the respective phase currents determines the instantaneous reactive power according to Eq.(64). For a closer analysis, e.g., for the switching status [011] the phase currents according to $i_{U,RST} = i_{U,RST,p} + i_{U,RST,q}$ have to be split into a part for the instantaneous real power and for the instantaneous reactive power. For the instantaneous power components we have (see Fig.13)

$$\begin{aligned} p_{U,[011]} &= (i_{U,S}\,u_{ZK} + i_{U,T}\,u_{ZK}) &&= (p_{U,S} + p_{U,T}) \\ q_{U,[011]} &= \tfrac{1}{\sqrt{3}}(i_{U,S}\,u_{ZK} - i_{U,T}\,u_{ZK}) &&= \tfrac{1}{\sqrt{3}}(p_{U,S} - p_{U,T})\,. \end{aligned} \tag{65}$$

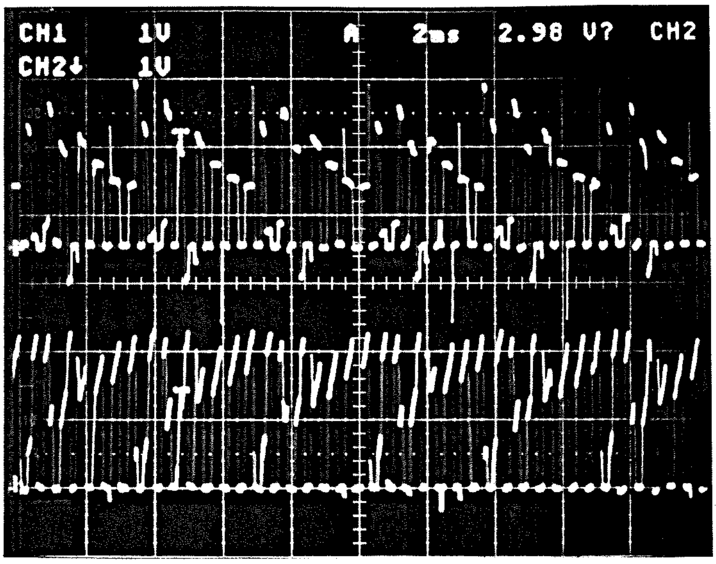

Fig.10. Instantaneous real and reactive power (p_U/U_{ZK} (lower trace) or q_U/U_{ZK} (upper trace), respectively) for a voltage DC link PWM converter system for supplying an induction machine under load conditions (c.f. Fig.1) related to an approximately constant DC link voltage U_{ZK}

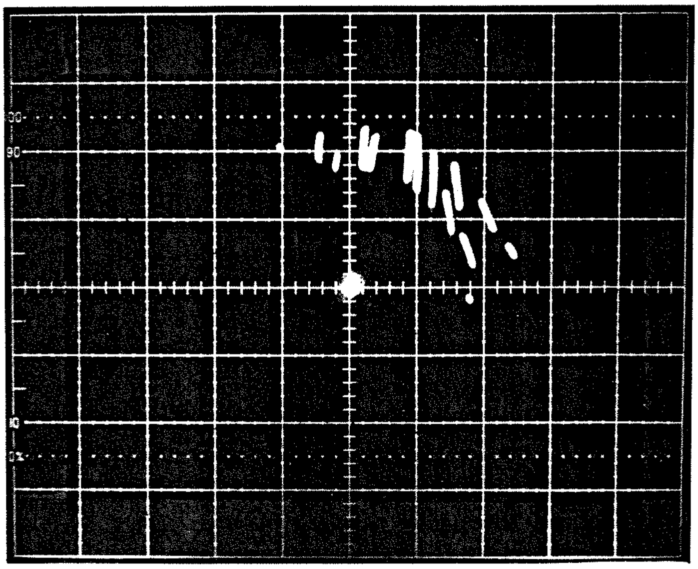

Fig.11. Representation of a space vector of the instantaneous power (trajectory) related to the DC link voltage (c.f. Fig.10; vertical: instantaneous real power; horizontal: instantaneous reactive power); for exclusive consideration of fundamentals (averaging) there follows a point of the circle diagram of the induction machine

The transformation of the instantaneous power components into phase currents (Eq.(46)) leads to

$$\begin{aligned} i_{U,R,p,[011]} &= -i_{U,R} \\ i_{U,S,p,[011]} &= \frac{(i_{U,S}+i_{U,T})}{2} \\ i_{U,T,p,[011]} &= \frac{(i_{U,T}+i_{U,S})}{2} \end{aligned} \tag{66}$$

and

$$\begin{aligned} i_{U,R,q,[011]} &= 0 \\ i_{U,S,q,[011]} &= \frac{(i_{U,S}-i_{U,T})}{2} \\ i_{U,T,q,[011]} &= \frac{(i_{U,T}-i_{U,S})}{2} = -i_{U,S,q,[011]} \; . \end{aligned} \tag{67}$$

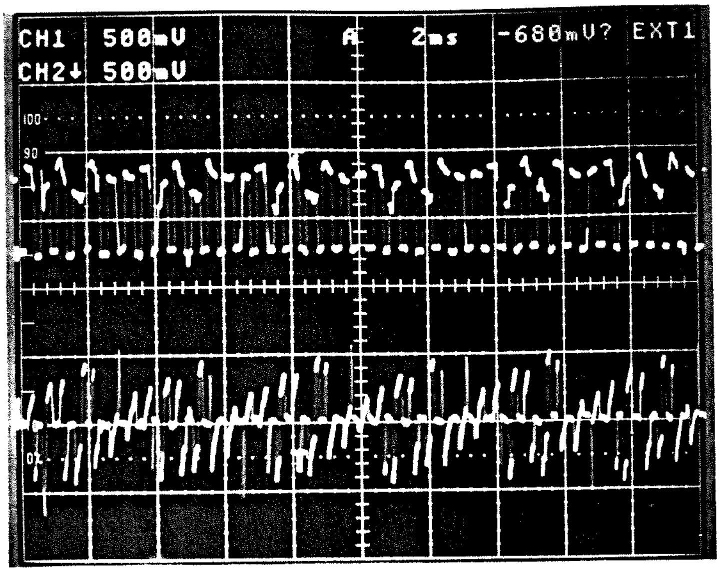

Fig.12. As Fig.10, but induction machine for no load condition; the mean value of the DC link current (close to zero) covers the machine no load losses

s_R	s_S	s_T	$\frac{1}{U_{ZK}} p_U(t)$	$\frac{1}{U_{ZK}} q_U(t)$
0	0	0	0	0
0	0	1	$i_{U,T}$	$\frac{1}{\sqrt{3}}(i_{U,S} - i_{U,R})$
0	1	0	$i_{U,S}$	$\frac{1}{\sqrt{3}}(i_{U,R} - i_{U,T})$
0	1	1	$i_{U,S} + i_{U,T} = -i_{U,R}$	$\frac{1}{\sqrt{3}}(i_{U,S} - i_{U,T})$
1	0	0	$i_{U,R}$	$\frac{1}{\sqrt{3}}(i_{U,T} - i_{U,S})$
1	0	1	$i_{U,R} + i_{U,T} = -i_{U,S}$	$\frac{1}{\sqrt{3}}(i_{U,T} - i_{U,R})$
1	1	0	$i_{U,R} + i_{U,S} = -i_{U,T}$	$\frac{1}{\sqrt{3}}(i_{U,R} - i_{U,S})$
1	1	1	0	0

Table 1

The current determining the instantaneous reactive power therefore is given by that current component which flows between the two shorted phases S and T. Thereby no resulting power flow is

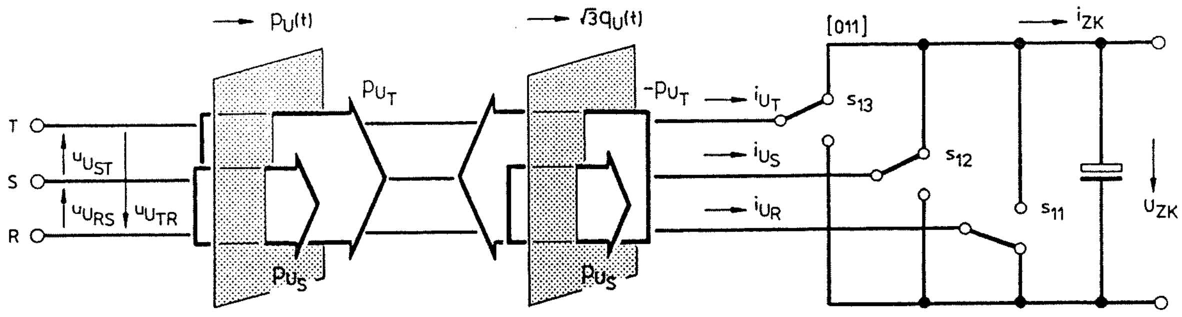

Fig.13. Power flow components of a voltage DC link PWM converter system for the switching status [011] ($s_{11} = 0$, $s_{12} = 1$, $s_{13} = 1$)

present. This also gives an illustrative description of the power flow between the phases which is connected to the instantaneous reactive power (see Fig.14).

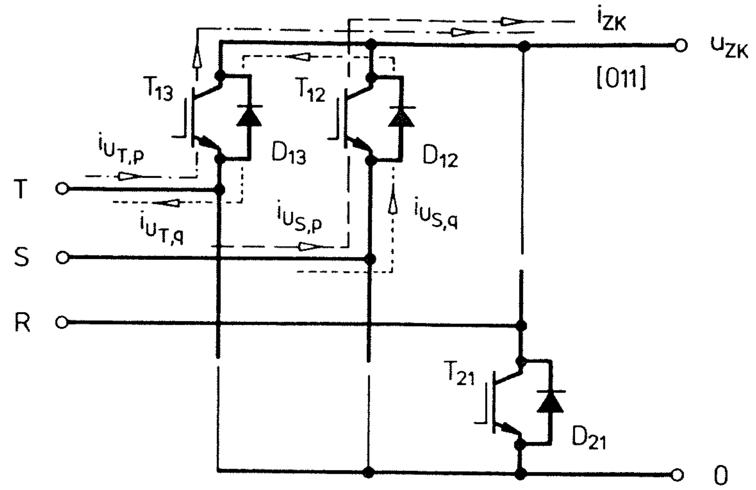

Fig.14. Illustration of the formation of an instantaneous reactive power under the presence of nonlinear elements (semiconductor devices of a voltage DC link PWM converter; switching status [011])

Naturally only the arithmetic mean of the currents of the shorted phases results in a contribution to the real power. As a coarse simplification it is possible to form an instantaneous reactive power based on a system nonlinearity (switching status dependent short circuit between two phases) also for interpreting a short by an infinitely large capacitance (which does not build up a voltage independent of the current flowing). We want to mention critically, however, that this formulation connects a quantity whose existence is related here to the nonlinear system behavior to an energy storage device which cannot exclusively cause the existence of an instantaneous reactive power.

Analysis of the Current DC Link Converter System [12], [14], [15], [16]

For the current DC link converter (see Fig.15) the converter current system is formed by an appropriate distribution of the (very much constant) DC link current to the different phases. This distribution is given by a proper control method. In turn, the system switching status is completely defined by a switching status matrix (Eq.(57)). However, concerning the phase switching functions the limitations

$$s_{11} + s_{12} + s_{13} = 1 \qquad s_{21} + s_{22} + s_{23} = 1 \tag{68}$$

have to be observed. This means that simultaneous conduction of more than one power electronic device in one bridge half has to be excluded if complete controllability is required. The operation of one bridge half therefore can be replaced by one three-pole switch between the phases concerning its function. If again based on

$$\underline{i}_U = \underline{s}\, I_{ZK} \tag{69}$$

a switching status space vector is defined, we have with

$$\begin{aligned} s_R &= \frac{1}{I_{ZK}} i_{U,R} &= (s_{11} - s_{21}) \\ s_S &= \frac{1}{I_{ZK}} i_{U,S} &= (s_{12} - s_{22}) \\ s_T &= \frac{1}{I_{ZK}} i_{U,T} &= (s_{13} - s_{23})\ , \end{aligned} \tag{70}$$

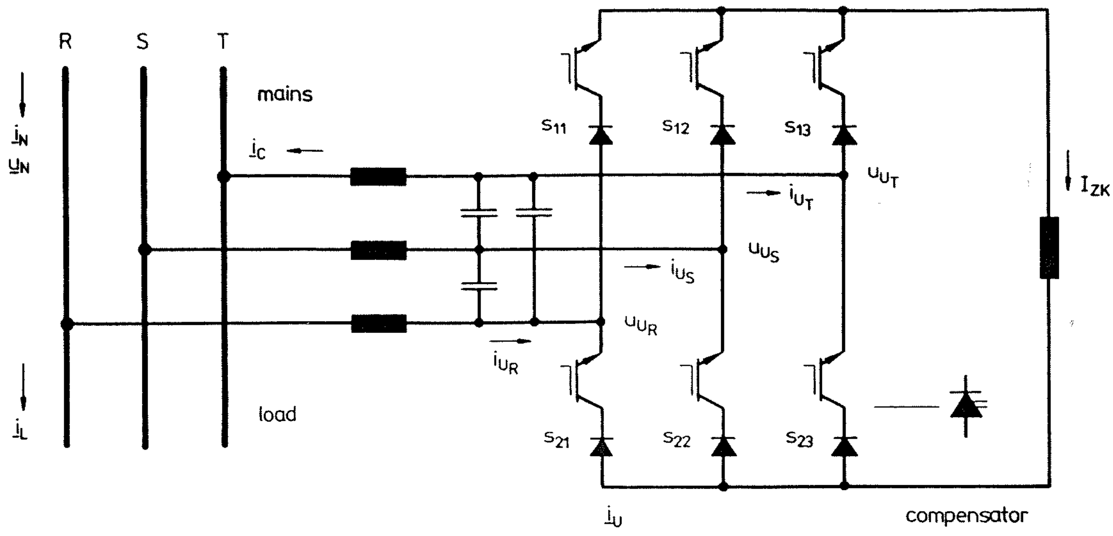

Fig.15. Realization of a compensator by a current DC link PWM converter system

$$\underline{s} = \frac{2}{3}\left[(s_{11}-s_{21}) + \underline{a}(s_{12}-s_{22}) + \underline{a}^2(s_{13}-s_{23})\right] \qquad |\underline{s}| = \frac{2}{\sqrt{3}}\,. \tag{71}$$

For generating the instantaneous real and reactive power components according to

$$\frac{p_U(t)}{I_{ZK}} = \frac{3}{2}\Re\left\{\underline{u}_U\,\underline{s}^*\right\}, \tag{72}$$

$$\frac{1}{I_{ZK}}p(t) = u_{ZK} = u_{U,R}\,(s_{11}-s_{21}) + u_{U,S}\,(s_{12}-s_{22}) + u_{U,T}\,(s_{13}-s_{23})\,, \tag{73}$$

$$\frac{q_U(t)}{I_{ZK}} = \frac{3}{2}\Im\left\{\underline{u}_U\,\underline{s}^*\right\}, \tag{74}$$

$$\frac{1}{I_{ZK}}q(t) = \frac{1}{\sqrt{3}}\Big[(u_{U,S}-u_{U,T})\,(s_{11}-s_{21}) + (u_{U,T}-u_{U,R})\,(s_{12}-s_{22}) + (u_{U,R}-u_{U,S})\,(s_{13}-s_{23})\Big] \tag{75}$$

again a circuit can be given (see Fig.16) whose structure corresponds to the power circuit of the PWM converter (and which avoids application of analog multipliers). There, however, all power electronic devices (therefore the whole converter structure) have to be represented by analog switches because the knowledge of only one switching function of a bridge leg is not sufficient for characterizing the switching status of the second valve of the same bridge leg (see Eqs.(68)). A summary of the instantaneous real and reactive power expressions valid for the different switching states is given by Table 2.

For a given converter switching state there is given an open-loop for at least one phase here. According to the duality of the current and voltage DC link PWM converter systems the phase voltage of this phase appears as forming the reactive power. The instantaneous power flow components given, e.g., for switching status [100,001] can be explained according to

$$\begin{aligned} p_{U,\,{}^{100}_{001}} &= (i_{ZK}\,u_{U,ST} + i_{ZK}\,u_{U,RS}) = (p_{U,ST} + p_{U,RS}) \\ q_{U,\,{}^{100}_{001}} &= \tfrac{1}{\sqrt{3}}(i_{ZK}\,u_{U,ST} - i_{ZK}\,u_{U,RS}) = \tfrac{1}{\sqrt{3}}(p_{U,ST} - p_{U,RS}) \end{aligned} \tag{76}$$

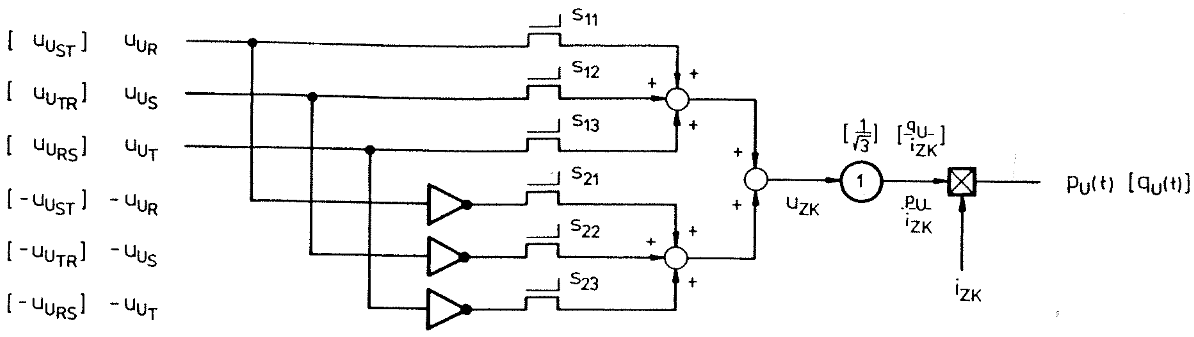

Fig.16. Determination of the instantaneous real and reactive power flow components of a current DC link PWM converter (for formation of the instantaneous reactive power the quantities in parantheses are valid)

(as already shown for the voltage DC link converter) by superposition of two power components of the phases conducting current here (see Fig.17).

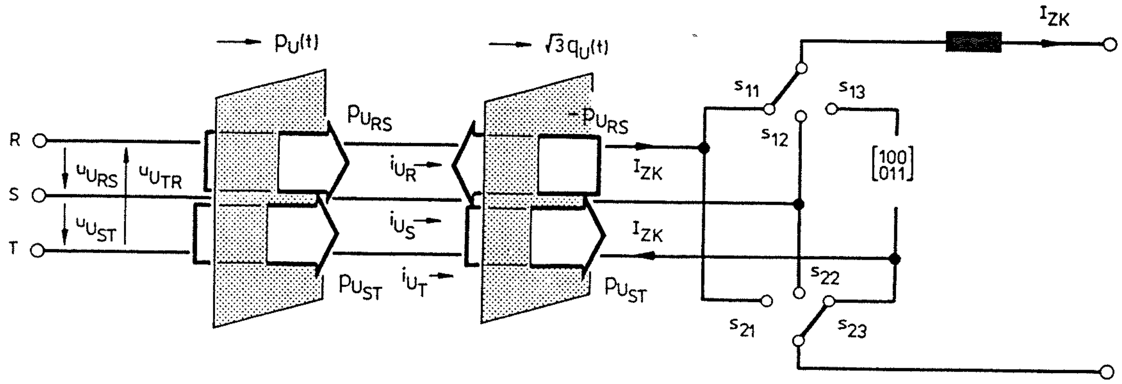

Fig.17. Power flow components of a current DC link PWM converter for switching status **S** = [100,001]

Instantaneous Power Components of Inductive and Capacitive Power Storage Devices [14]

Finally, as a simple application of the theory described here so far we shall briefly discuss the characterization of a symmetrical magnetic or electrical storage device by the instantaneous real and reactive power flow.

For the space vector of the instantaneous power of an inductive storage device we have in analogy to Eq.(17)

$$\underline{s}_L(t) \quad = \quad p_L(t) + jq_L(t) \; . \tag{77}$$

$\mathbf{S}$	$\frac{1}{I_{ZK}} p_U(t)$	$\frac{1}{I_{ZK}} q_U(t)$
$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$		
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	0	0
$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$		
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$(u_{U,R} - u_{U,T})$	$\frac{1}{\sqrt{3}}(u_{U,ST} - u_{U,RS}) = +\sqrt{3}u'_{U,S}$
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	$(u_{U,R} - u_{U,S})$	$\frac{1}{\sqrt{3}}(u_{U,ST} - u_{U,TR}) = -\sqrt{3}u'_{U,T}$
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$(u_{U,S} - u_{U,T})$	$\frac{1}{\sqrt{3}}(u_{U,TR} - u_{U,RS}) = -\sqrt{3}u'_{U,R}$
$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	$(u_{U,S} - u_{U,R})$	$\frac{1}{\sqrt{3}}(u_{U,TR} - u_{U,ST}) = +\sqrt{3}u'_{U,T}$
$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$(u_{U,T} - u_{U,S})$	$\frac{1}{\sqrt{3}}(u_{U,RS} - u_{U,TR}) = +\sqrt{3}u'_{U,R}$
$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$	$(u_{U,T} - u_{U,R})$	$\frac{1}{\sqrt{3}}(u_{U,RS} - u_{U,ST}) = -\sqrt{3}u'_{U,S}$

Table 2

From Eq.(77), under consideration of

$$\underline{i}_L = |\underline{i}_L| \exp j\varphi_{\underline{i}_L} \qquad \underline{u}_L = L\frac{d\underline{i}_L}{dt} \tag{78}$$

there follows

$$\underline{s}_L(t) = \frac{3}{2}\left\{L|\underline{i}_L|\frac{d|\underline{i}_L|}{dt} + j|\underline{i}_L|^2\omega_{\underline{i}_L}L\right\} \qquad \frac{d\varphi_{\underline{i}_L}}{dt} = \omega_{\underline{i}_L} \; . \tag{79}$$

The instantaneous real power

$$p_L(t) = \frac{d}{dt}\left[\frac{1}{2}L\left(\frac{3}{2}|\underline{i}_L|^2\right)\right] = \frac{d}{dt}w_{mag}(t) \tag{80}$$

only is present when the total magnetic energy of the storage device changes, i.e., when the magnitude of the current space vector changes. The instantaneous reactive power is dependent on the stored magnetic energy and on the angular speed of the space vector according to

$$q_L(t) = \frac{3}{2}|\underline{i}_L|^2\omega_{\underline{i}_L}L = 2\omega_{\underline{i}_L}\, w_{mag}(t) \; . \tag{81}$$

Dynamically, by no means it always assumes only a positive value. A rotation of a current vector of constant magnitude therefore is (c.f. Eq.(80)) possible "without resistance"; that means that this is not connected with a resulting energy exchange between storage device and mains. The angular speed of the current space vector therefore only determines the rate of re-distribution of the phase energies of the storage device by corresponding currents which flow through the supplying mains. For the stationary symmetric case (current space vector of constant magnitude and constant angular speed) we therefore have

$$Q_L = \frac{3}{2}\omega L\hat{I}_L^2 = 3\omega L I_{L,rms}^2 \qquad P_L = 0 \; . \tag{82}$$

The instantaneous power components then turn into the quantities P_L and Q_L (known from basic electrical engineering). There, Q_L shows a positive value here (i.e., for the magnetic storage) according to the definition of the instantaneous power space vector (Eq.(17)).

For considering a capacitive storage device there follow with

$$\underline{s}_C(t) = p_C(t) + jq_C(t) \tag{83}$$

and

$$\underline{u}_C = |\underline{u}_C| \exp j\varphi_{\underline{u}_C} \qquad \underline{i}_C = C\frac{d\underline{u}_C}{dt} \tag{84}$$

these completely analogous relations:

$$p_C(t) = \frac{d}{dt}\left[\frac{1}{2}C\left(\frac{3}{2}|\underline{u}_C|^2\right)\right] = \frac{d}{dt}w_{el}(t) \tag{85}$$

$$q_C(t) = -\frac{3}{2}|\underline{u}_C|\,\omega_{\underline{u}_C} C = -2\omega_{\underline{u}_C}\, w'_{el}(t) \tag{86}$$

The sign of Q_C is negative for the stationary, symmetric case!

Conclusions

This paper treats the description of a three-wire system by space vector calculus concerning the appearing instantaneous power flow components. The quantities *instantaneous real power* and *instantaneous reactive power* are defined. For disappearing instantaneous reactive power a power loss optimal energy transmission is given. Therefore, an immediate meaning is given for this quantity.

One special advantage of space vector calculus is its clearness (because the representation can be made with minimum effort). Furthermore, its application to the control of a converter suggests itself; this can be said due to the possibility of direct inclusion of the quantities instantaneous real and instantaneous reactive power (which are determined via a relation of voltage and current space vector components) into the control of the power electronic system. The reason for inclusion of the quantities mentioned is given by the fact that a power electronic system finally has to define the instantaneous power flow components.

Of special importance furthermore is the fact that for voltage and current DC link PWM converter systems the instantaneous power components simply can be determined by a representation of the converter function with little effort concerning circuit technology. (The PWM converter system mentioned represents that group of forced commutated converters which have many applications especially in the drives area.)

Generally it has to be mentioned that the considerations given here naturally form only a part of a general theory which still has to be analyzed in more detail. It has been tried here to describe this part of the theory in relation to power conversion in power electronic systems.

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