Efficiency Optimization of a 100-W 500 000-r/min Permanent-Magnet Machine Including Air-Friction Losses

Jorma Luomi, Member, IEEE, Christof Zwyssig, Student Member, IEEE, Andreas Looser, Student Member, IEEE, and Johann W. Kolar, Senior Member, IEEE

Abstract—This paper proposes a method for the efficiency optimization of ultrahigh-speed permanent-magnet machines. Analytical methods are applied for the modeling of the machine that is equipped with a diametrically magnetized rotor and a slotless stator. The outer dimensions of the machine are design constraints, and the internal dimensioning is optimized for minimum losses. The air-friction losses are taken into account in addition to the usual iron, copper, and eddy-current losses. Laminated silicon–iron or laminated amorphous iron is used as the stator core material. The results show that air-friction losses influence the optimum design considerably, leading to a small rotor diameter at high speeds. The loss minimization and the amorphous iron core make it possible to reduce the calculated losses by 63% as compared to a machine design not considering air-friction losses. The resulting efficiency is 95% for a 100-W 500 000-r/min machine excluding bearing losses. Experimental results are shown to illustrate the validity of the method.

Index Terms—Air friction, high speed, optimization, permanent magnet (PM).

I. INTRODUCTION

ULTRAHIGH-SPEED electrical-drive systems are developed for new emerging applications, such as generators/starters for micro gas turbines, turbo-compressor systems, drills for medical applications, and spindles for machining. Typically, the power ratings of these applications range from a few watts to a few kilowatts and the speeds from a few tens of thousands revolutions per minute up to a million revolutions per minute [1]. Recently, a 100-W 500 000-r/min permanent-magnet (PM) machine has been designed and investigated experimentally [2], [3]. For such high speeds, the mechanical-rotor construction and the minimization of high-frequency losses are the main challenges. This machine has a diametrically magnetized cylindrical Sm<sub>2</sub>C<sub>0.17</sub> PM encased in a titanium sleeve for sufficiently low mechanical stresses on the magnet. The slotless stator core consists of 168-μm silicon–iron laminations, and the three-phase air-gap winding is made of litz wire with 0.071-mm strands for low copper losses. The cross section of the machine is shown in Fig. 1.

In designing an ultrahigh-speed machine, it is important to optimize the efficiency—i.e., minimize the losses—when the outer dimensions of the machine are design constraints. Previously, such optimizations have been based on resistive losses in the stator winding and iron losses in the stator core [4] and, in addition, eddy-current losses in the rotor [5]. However, air-friction losses are an important part of the total losses in an ultrahigh-speed machine [2]. Therefore, these losses should also be taken into account in the optimization.

This paper proposes a method for the loss minimization of ultrahigh-speed PM machines including air-friction losses. Analytical methods are applied for the modeling. In the following, models are first presented for the magnetic field and the loss components. Then, the optimization procedure is briefly described, and results of the loss minimization are shown. Finally, experimental results are presented.

II. MAGNETIC FIELD AND TORQUE

A solution of the magnetic field is required for analyzing the operating point, resistive losses, eddy-current losses, and iron losses. Today, numerical analysis based on the finite-element
The frequency of the stator current is high (8.3 kHz in the 500 000-r/min machine). Therefore, eddy currents increase the copper losses of the stator winding. In addition to the stator current, the air-gap flux causes considerable eddy-current losses in the winding due to the slotless design of the stator. In order to reduce the losses, the winding is made of litz wire.

The copper losses consist of the current-dependent resistive losses \( P_{\text{Cu},s} \) in the stator winding, which include the influence of the skin effect and of the proximity-effect losses \( P_{\text{Cu},p} \), which are mainly due to the eddy currents induced by the magnetic field of the PM. The copper losses are

\[
P_{\text{Cu}} = P_{\text{Cu},s} + P_{\text{Cu},p} = I^2 R + G \left( \frac{H}{\sigma} \right)^2
\]

where \( I \) is the rms stator current, \( \bar{H} \) is the peak magnetic field strength in the winding, \( \sigma \) is the conductivity of the conductor, and the coefficients \( F \) and \( G \) include the effects of the eddy currents. The Ferreira method was chosen for the modeling of the eddy-current effects; the coefficients \( F \) and \( G \) are calculated based on the frequency, the conductivity, and the geometry of the winding arrangement, as described in [14]. At significantly higher frequencies or larger strand diameters, the accuracy could be increased by using a method based on function fitting for the calculation of the proximity-effect losses [15].

### B. Iron Losses

The iron losses are calculated as an integral over the iron volume \( V_{\text{Fe}} \) using the Steinmetz equation

\[
P_{\text{Fe}} = \int_{V_{\text{Fe}}} C_m \cdot f^\alpha \cdot \bar{B}^\beta \, dV
\]

where \( f \) is the frequency and \( \bar{B} \) is the peak magnetic flux density. The coefficients \( C_m \), \( \alpha \), and \( \beta \) are taken from the manufacturer’s data sheets for the frequency range to be considered.

### C. Air-Friction Losses

For simple geometries, such as cylinders and disks, air-friction losses can be calculated analytically with friction coefficients based on empirical data [16]. In the following, only the air gap is taken into account in the calculation of the air-friction losses, and the losses at the end caps are omitted.

The air-friction losses of a long rotating cylinder encased in a stationary hollow cylinder are

\[
P_{\text{f,air}} = \frac{\rho_{\text{air}} \pi \omega^3 R_2^4 L}{c_f \nu}
\]

where \( \rho_{\text{air}} \) is the density of the air, \( \omega \) is the angular speed, \( R_2 \) is the radius of the cylinder, and \( L \) is the length of the cylinder. The friction coefficient \( c_f \) depends on the radius of the cylinder, the air gap \( \delta \), and the Reynolds and the Taylor numbers, which are defined as

\[
Re = \frac{R_2^2 \omega}{\nu}, \quad Ta = \frac{R_2 \omega \delta}{\nu \sqrt{\frac{\delta}{R_2}}}
\]

where \( \nu \) is the kinematic viscosity of air. The flow stability depends on the Taylor number; the flow can be divided into laminar Couette flow (\( Ta < 41.3 \)), laminar flow with Taylor vortices (\( 41.3 < Ta < 400 \)), and turbulent flow (\( Ta > 400 \)).
For laminar Couette flow, the friction coefficient can be determined analytically, but measurements show discrepancies with the theoretical values. Therefore, empirical data are usually used, and correction factors are applied to adapt for different geometries [16]. For the air gap of the machine under investigation, the friction coefficient

\[ c_f = \frac{1.8}{Re} \left( \frac{\delta}{R_2} \right)^{0.25} \frac{R_2^3}{R_2^3 - R_2^2} \]  

(5)
can be used. Beyond the transition point from laminar flow to flow with Taylor vortices, measurements show a friction coefficient

\[ c_f \propto Ta^{-0.2} \]  

(6)

This model for air-friction losses has been experimentally validated in [16].

D. Other Losses and Shaft Torque

The nonsinusoidal distribution of the stator winding causes spatial harmonics in the magnetic field, and time-harmonics are present in the stator voltage due to the inverter supply, but the eddy-current losses in the rotor of a slotless PM machine are generally very low, as shown in [5]. Therefore, the rotor losses are ignored in the following. Furthermore, it is to be noted that the bearing losses are not considered in the efficiency optimization of the machine, since they are usually more dependent on the application than on the inner dimensions of the motor.

The machine is to be optimized for a given mechanical rotor torque \( T_m \). When the proximity-effect, iron, and air-friction losses are taken into account, the electromagnetic torque can be calculated by

\[ T_e = T_m + \frac{P_{Cu,p} + P_{Fe} + P_{f,air}}{\omega} \]  

(7)

IV. MECHANICAL MODEL

A 2-D mechanical model is used for the rotor. The stresses of the rotor construction with the PM shrink fitted into a titanium sleeve have been analyzed in [3]. The following specifications have to be fulfilled in the entire operation region.

1) The torque transfer and low eccentricity are guaranteed by allowing no lift off of the sleeve. Thus, the radial stress at the interface between the PM and the sleeve has to be negative, which is most critical at the maximum speed.

2) Stresses in the entire PM have a safety margin of 30% to the tensile strength of Sm\(_2\)Co\(_{17}\) (120 MPa). The most critical stress occurs at the maximum speed in the center of the magnet.

3) Stresses in the entire sleeve have a safety margin of 50% to the tensile strength of titanium (900 MPa). The most critical stress occurs at the maximum speed on the inner side of the sleeve.

4) The sleeve has a minimum thickness (0.25 mm) for manufacturability reasons.

V. OPTIMIZATION

A. Loss Minimization

The goal is to minimize the total losses obtained from (1)–(3), i.e., the objective function is

\[ P_d = P_{Cu} + P_{Fe} + P_{f,air}. \]  

(8)

The losses are minimized for a given rotational speed \( n \) and shaft torque \( T_m \). The outer radius \( R_o \) and length \( L \) of the stator core are kept constant (or changed in an outer iteration). The independent variables are the magnet radius \( R_1 \), the air gap \( \delta \), and the inner radius of the stator core \( R_4 \).

The loss minimization is constrained in order to obtain a geometrically, mechanically, and magnetically feasible design. The sleeve thickness \( R_2 - R_1 \) is kept at the minimum value given by the mechanical analysis. The minimum value for the air gap \( \delta \) is 0.2 mm, and the minimum value for the thickness of the stator core \( (R_5 - R_4) \) is 1 mm. In addition, the flux density in the iron core is limited to a maximum value (1.3 T for a silicon–iron stack and 1.1 T for an amorphous iron stack). In the results presented in this paper, a constant temperature of 120 °C is assumed for the stator winding.

Many different methods can be used for solving the minimization problem. A straightforward choice is the Nelder–Mead simplex method included in the MATLAB software as the function fminsearch. The constraints can be included in this derivative-free minimization method by giving the objective function a high value if the design is not feasible.

B. Litz-Wire Optimization

The strand diameter of the litz wire influences only the copper losses. During the loss minimization, the strand diameter giving the lowest copper losses is selected for every feasible design. Making the strands thinner decreases the eddy currents but increases the resistive losses if the winding fill factor decreases.

The winding fill factor is given by

\[ k_{Cu} = k_{Cu,t}k_{Cu,s}. \]  

(9)

Here, the turn fill factor \( k_{Cu,t} \) is defined as the ratio of the area occupied by the litz wires to the total cross-sectional area of the winding, and the strand packing factor \( k_{Cu,s} \) is the ratio of the copper area of the strands in the wire to the area of the wire. The turn fill factor is assumed to be constant, whereas the strand packing factor is a function of the strand diameter. Fig. 2 shows the strand packing factors \( k_{Cu,s} \) for various strand diameters obtained from the manufacturer’s data [17].

VI. RESULTS

A. Mechanical-Rotor Model

The thickness of the rotor sleeve and the interference fit required were calculated using the mechanical model. The results are shown in Fig. 3 for a rotor with a maximum speed of 500,000 r/min. For small rotor radii, the sleeve thickness is
Fig. 2. Strand packing factor $k_{Cu,s}$ as function of strand diameter. (Markers) Manufacturer’s data. (Line) Fitted curve.

Fig. 3. Results of mechanical analysis as function of rotor radius at 500 000 r/min. The first subfigure shows the sleeve thickness $R_2 - R_1$, the second subfigure shows the shrink fit, and the third subfigure shows (solid) radial stress in the center of the PM, (dashed) radial stress at the interface ($R_1$), (dash-dotted) tangential stress in the sleeve at $R_1$, and (dotted) stress limits in the magnet and sleeve.

at the minimum value defined by the manufacturability; the shrink fit ensures that the sleeve does not lift off. At 3-mm magnet radius, the stress in the magnet reaches its limit, and the shrink fit has to be enforced to guarantee the safety margin to the tensile strength of Sm$_2$Co$_{17}$. At 3.3 mm, the stress in the titanium sleeve reaches its limit, and the sleeve thickness has to be increased in order to guarantee the safety margin to the tensile strength of titanium.

**B. Fixed Outer Dimensions**

The parameters used in the efficiency optimization are given in Table I. Figs. 4 and 5 show the dependence of the losses on the internal radial dimensions of the machine. The outer dimensions of an existing machine were used ($R_5 = 8$ mm and $L = 15$ mm [3]), and the stator core material was laminated silicon–iron. The sleeve thickness was fixed to the original value (0.5 mm) and so were the air gap (0.5 mm) and the strand diameter of the litz wire (0.071 mm). In Fig. 4, the total losses are shown as a function of the magnet radius $R_1$ for various values of the inner radius $R_4$ of the stator core. The circle shows the value for the existing machine with $R_4 = 5.5$ mm and $R_1 = 2.5$ mm [3].

<table>
<thead>
<tr>
<th><strong>TABLE I</strong></th>
<th><strong>PARAMETERS USED IN EFFICIENCY OPTIMIZATION</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Symbol</strong></td>
<td><strong>Quantity</strong></td>
</tr>
<tr>
<td>$B_{rem}$</td>
<td>Remanence flux density</td>
</tr>
<tr>
<td>$\mu_{r1}$</td>
<td>Relative recoil permeability</td>
</tr>
<tr>
<td>$\mu_{rS}$</td>
<td>Relative permeability</td>
</tr>
<tr>
<td>$C_{m0}$</td>
<td>Steinmetz coefficient</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Steinmetz coefficient</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Steinmetz coefficient</td>
</tr>
<tr>
<td>$\rho_{air}$</td>
<td>Density</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
</tr>
</tbody>
</table>

Fig. 4. Losses of the machine with fixed outer dimensions of the stator core ($R_5 = 8$ mm, $L = 15$ mm) and a shaft power of 100 W at a rotational speed of 500 000 r/min for variable magnet radius $R_1$ and various values of the inner radius $R_4$ of the stator core. The circle shows the value for the existing machine with $R_4 = 5.5$ mm and $R_1 = 2.5$ mm [3].
it has 14.2 W of losses. Thus, a reduction of losses by 5.2 W can be obtained by changing the internal radial dimensions of the machine. In this case, the loss reduction is mainly due to the optimization of the magnet radius $R_4$. The loss reduction achieved by changing the inner radius $R_4$ of the stator core is about 0.01 W and, thus, negligible.

The optimization leads to a rather expected result: Decreasing the rotor diameter leads to a reduction of the air-friction losses. The iron losses are also reduced, since a smaller magnet radius leads to a lower air-gap flux density in the machine considered. In order to keep the torque constant, the total current of the stator winding has to be increased, leading to increasing copper losses when the magnet radius is decreased. However, a larger amount of air-friction and iron losses can be avoided by decreasing the magnet radius.

The losses of the existing machine obtained with the loss models presented in this paper are shown by circular markers in Figs. 4 and 5. There are small differences between the values shown in this paper and the ones obtained in [2] by means of measurements and separation of losses; these differences originate from the less accurate model used earlier for iron losses and from a small difference in the winding geometry and temperature.

C. Variable Outer Dimensions and Strand Diameter

For improved loss minimization, the sleeve thickness was reduced to the minimum value given by the mechanical analysis ($R_2 - R_1 = 0.25$ mm), and the optimization was based on the three independent variables ($R_1$, $\delta$, and $R_4$). In addition, the strand diameter giving lowest copper losses was determined for the litz wire. In all of the following examples, the loss minimization resulted in an air gap value of $\delta = 0.2$ mm, i.e., the minimum value. The optimum strand diameter of the litz wire varied between 0.03 and 0.05 mm, but the influence of small changes in the strand diameter is insignificant, since the proximity-effect losses are much lower than the other loss components.

Fig. 6 shows the magnet radius $R_1$ and inner radius $R_4$ of the stator core for the laminated silicon–iron core material. The results are shown for variable outer radius $R_5$ of the stator core and various values of the core length $L$. The shaft power is 100 W and the rotational speed 500 000 r/min.

D. Influence of Stator Core Material

Figs. 8 and 9 show the loss-minimization results when the stator core material was laminated amorphous iron (Megalas magnetic alloy 2605SA1). In this case, the iron losses are lower than 10% of the total losses. If the original outer dimensions of the machine are used, a loss reduction to about 5.2 W is possible by choosing $R_4 = 5.1$ mm and $R_1 = 1.8$ mm. Thus, the reduction of the air-gap and sleeve thickness allows a loss reduction by 2 W from the result shown in Fig. 4. The contributions of the air-gap and sleeve thickness to this loss reduction are 0.7 and 1.3 W, respectively. The losses can be further reduced by increasing the outer dimensions of the machine.
Fig. 8. Optimization results for the amorphous iron core. Magnet radius $R_1$ and inner radius $R_4$ of the stator core for variable outer radius $R_5$ of the stator core and various values of the core length $L$. The shaft power is 100 W and the rotational speed 500,000 r/min.

or larger. Thus, the outer diameter of the stator can be reduced from the original value with almost no influence on the losses. The losses are higher at the lowest values of $R_5$. The loss increase is caused by two constraints used in the optimization: the minimum thickness of the stator core (1 mm) and the maximum flux density allowed in the stator core (1.1 T).

E. Influence of Air-Friction Losses

The air-friction losses (3) are approximately proportional to $R_4^2$. At high speeds, the inclusion of this loss component in the loss minimization leads to a smaller rotor radius than the one obtained without this loss component. This fact is shown in Figs. 10 and 11 showing the loss-minimization results when the air-friction losses are omitted.

F. Influence of Speed

In order to investigate the influence of the rotational speed on the results, the losses of 100-W motors were minimized in the speed range between 100,000 and 1,000,000 r/min. The core length was fixed to $L = 15$ mm, and the outer radius $R_5$ of the core was adjusted in such a way that the flux density was 1.1 T (if possible without contradicting the minimum-core-thickness constraint). The results are shown in Figs. 12 and 13. It is obvious that the inclusion of the air-friction losses in the loss minimization leads to very small rotor diameters at the highest speeds. For the power rating considered, the analysis of rotor dynamics would have to be included in the optimization for speeds higher than 500,000 r/min, which finally restricts the reduction of the rotor radius with increasing speed.
VII. EXPERIMENTAL RESULTS

A. Hardware

In order to validate the models and the optimization method, experiments were carried out using two machines that were built during this paper. The first machine (Machine A) is shown in Fig. 14. It was designed without the proposed efficiency-optimization method that includes air-friction losses, but a litz-wire winding and an amorphous iron core were used. The 100-W 500 000-r/min machine is used as generator in a miniature-gas-turbine application and for driving an ultracompact turbo compressor [18], [19].

The second machine (Machine B) is shown in Fig. 15. It was designed for a rotational speed of 1 000 000 r/min and a shaft power of 100 W, using the efficiency-optimization method that includes air-friction losses. Both machines are manufactured equally and have rotors that are supported by the same high-speed ball bearings. The bearing selection and losses are not part of the analysis in this paper as the bearing losses are independent of the optimization parameters, and the bearing selection depends strongly on the application.

B. Measurements

It is not easy to measure the losses of a machine having a very low torque and a very high speed (in the millinewton-meter region and above 100 000 r/min), the loss measurement being a field of research itself [20], [21]. A rotary torque transducer is not feasible due to the high speed, and reaction torque measurement of very small torque differences offers insufficient accuracy. Therefore, the deceleration test is the preferred method [16]. For the measurements, the rotor was accelerated to a certain speed, after which the drive system was switched off, letting the losses decelerate the rotor. The speed was recorded as a function of time, and the losses were calculated afterward using the known moment of inertia of the rotor.

The measured losses include the proximity-effect, iron, air-friction, and bearing-friction losses. The current-dependent resistive losses $P_{Cu,s}$ are excluded, but they can be estimated very accurately by using the measured dc resistance, since the skin effect is negligible due to the litz wire. The bearing-friction losses are included in the measurement but not included in the total losses $P_d$ that are calculated in the optimization procedure. Estimating the friction losses of ball bearings is difficult because these losses depend on the age of the bearings, the axial preload, the temperature, and the unbalance of the rotor.

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The measured and calculated losses of Machine A are shown in Fig. 16, and the corresponding losses of Machine B are shown in Fig. 17. It is to be noted that the calculated losses do not include the bearing-friction losses. The manufacturer’s estimate is 5 W for the two bearings at 500 000 r/min, which corresponds well to the results shown in Figs. 16 and 17. It can be seen that the losses can be drastically reduced by increasing the rotor diameter. This reduction is mainly due to decreased air-friction losses, which are proportional to the power of four of the rotor radius in (3).
material—such as amorphous iron—allows for a smaller outer diameter of the stator with an insignificant increase in the total losses. Deceleration measurements of existing prototypes proved that the losses in ultrahigh-speed operation can be drastically reduced by decreasing the rotor diameter, mainly due to the lower air-friction losses.

Increasing the speed toward 1 000 000 rpm results in machines with decreasing magnet, shaft, and stator radii. For designing machines to be integrated into various applications, the efficiency optimization has to be coupled with the rotor dynamic and thermal analyses. Results of research in this direction will be published in a future paper.

**TABLE II**

<table>
<thead>
<tr>
<th>Change of</th>
<th>Old</th>
<th>New</th>
<th>Loss reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet radius (R_1)</td>
<td>2.5 mm</td>
<td>1.7 mm</td>
<td>5.2 W</td>
</tr>
<tr>
<td>Inner radius (R_2) of stator core</td>
<td>5.5 mm</td>
<td>5.3 mm</td>
<td>0 W</td>
</tr>
<tr>
<td>Air gap (R_3 - R_2)</td>
<td>0.5 mm</td>
<td>0.2 mm</td>
<td>0.7 W</td>
</tr>
<tr>
<td>Sleeve thickness (R_2 - R_1)</td>
<td>0.5 mm</td>
<td>0.25 mm</td>
<td>1.3 W</td>
</tr>
<tr>
<td>Iron core material</td>
<td>SiFe</td>
<td>amorphous</td>
<td>1.8 W</td>
</tr>
</tbody>
</table>

**APPENDIX**

A. Magnetic Field Problem Formulation

A 2-D boundary-value problem is formulated for the magnetic field, and the effects of the third dimension are ignored. The polar \(r-\theta\) coordinate system fixed to the rotor cross section is shown in Fig. 1. The diametrically magnetized PM has a uniformly distributed remanence flux density \(B_{\text{rem}}\) in the direction of \(\theta = 0\). The magnetic flux density is given by

\[
B = \mu_0 M_p + \mu_r \mu_0 H \tag{10}
\]

where \(M_p\) is the permanent magnetization, \(H\) is the magnetic field strength, and \(\mu_r\) is the relative recoil permeability.

The problem region is divided into three subregions. In the PM region \((0 \leq r \leq R_1)\), \(M_p = B_{\text{rem}}/\mu_0\), and \(\mu_r = \mu_{r1}\). The uniform permanent magnetization is given by

\[
M_p = u_r M_p \cos \theta - u_\theta M_p \sin \theta \tag{11}
\]

where \(u_r\) and \(u_\theta\) are the radial and azimuthal unit vectors, respectively. In the nonferromagnetic region between the PM and the stator core \((R_1 < r < R_4)\), \(M_p = 0\), and \(\mu_r = 1\). In the stator core \((R_4 < r < R_5)\), \(M_p = 0\), and \(\mu_r = \mu_{r5}\).

The magnetic field is modeled by means of the magnetic scalar potential \(\phi\) defined by \(H = -\nabla \phi\). Inserting this definition with (10) into the governing equation \(\nabla \cdot B = 0\) yields the Laplace equation \(\nabla^2 \phi = 0\) for the scalar potential. It is to be noted that \(\nabla \cdot M_p = 0\) for uniform permanent magnetization.

In the polar-coordinate system, the partial differential equation of the scalar potential is

\[
\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0. \tag{12}
\]

In addition to the partial differential equation, interface and boundary conditions have to be defined. The continuity of the tangential component of the magnetic field strength requires that \(\phi\) is continuous over the interfaces at \(R_1\) and \(R_4\). The continuity of the normal component of the magnetic flux density requires that \(M_p \cos \theta - \mu_r \partial \phi/\partial r\) is continuous over the interfaces at \(R_1\) and \(R_4\).

The normal component of the magnetic flux density vanishes at the outer boundary of the machine, which gives the boundary condition \(-\mu_r \partial \phi/\partial r = 0\) at \(R_5\).
B. Magnetic Field Solution

Expressions for the magnetic field are obtained in the whole machine by solving the problem consisting of the partial differential equation (12) and the interface and boundary conditions. The solution can be obtained by separation of variables. In the PM \((0 \leq r \leq R_1)\), the radial and azimuthal components of the magnetic flux density are

\[
B_r = K_{B1} \cos \theta \quad B_\theta = -K_{B1} \sin \theta
\]

(13)

respectively, where the flux density coefficient is

\[
K_{B1} = \frac{B_{\text{rem}}}{N} \left[ 1 - \frac{R_2^4}{R_5^4} \right] \left[ 1 + \frac{R_2^2}{R_4^2} \right]
+ \frac{1}{\mu_5} \left[ 1 + \frac{R_4^2}{R_5^2} \right] \left[ 1 - \frac{R_2^2}{R_4^2} \right]
\]

(14)

with the definition

\[
N = \left[ 1 - \left( \frac{R_4}{R_5} \right)^2 \right] \left[ (\mu_{r1} + 1) - (\mu_{r1} - 1) \left( \frac{R_1}{R_4} \right)^2 \right]
+ \frac{1}{\mu_5} \left[ 1 + \left( \frac{R_4}{R_5} \right)^2 \right] \left[ (\mu_{r1} + 1) + (\mu_{r1} - 1) \left( \frac{R_1}{R_4} \right)^2 \right].
\]

(15)

In the nonferromagnetic region \((R_1 \leq r \leq R_4)\), the radial and azimuthal components of the magnetic flux density are

\[
B_r = K_{B2} \left[ 1 + \left( \frac{R_4}{r} \right)^2 \right] \cos \theta
\]

\[
B_\theta = -K_{B2} \left[ 1 - \left( \frac{R_4}{r} \right)^2 \right] \sin \theta
\]

(16)

respectively, where the flux density coefficient is

\[
K_{B2} = \frac{B_{\text{rem}}}{N} \left[ 1 - \left( \frac{R_4}{R_5} \right)^2 \right] \left[ 1 - \frac{1}{\mu_5} \left( \frac{R_4}{R_5} \right)^2 \right] \left( \frac{R_1}{R_4} \right)^2.
\]

(17)

In the stator core \((R_4 \leq r \leq R_5)\), the components of the magnetic flux density are

\[
B_r = K_{B3} \left[ -1 + \left( \frac{R_5}{r} \right)^2 \right] \cos \theta
\]

\[
B_\theta = -K_{B3} \left[ 1 + \left( \frac{R_5}{r} \right)^2 \right] \sin \theta
\]

(18)

respectively, where the flux density coefficient is

\[
K_{B3} = \frac{2B_{\text{rem}}}{N} \left( \frac{R_1}{R_5} \right)^2.
\]

(19)

C. Torque Calculation

The density of the azimuthal-force component caused by the spatial fundamental wave \(J_1\) of the current density in the stator winding is \(J_1 B_r\). The electromagnetic torque is obtained as the integral

\[
T_e = L \int_{-\pi}^{\pi} r^2 J_1 B_r dr d\theta.
\]

(20)

The stator-current component in the direction of the PM flux is controlled to zero. Using (16), the integration in (20) results in

\[
T_e = \sqrt{2}\pi k_w k_{\text{Cu}} J K_{B2} L R_4^3 \left( \frac{4}{3} - \frac{R_3}{R_4} - \frac{1}{3} \frac{R_3^2}{R_4^3} \right)
\]

(21)

where \(k_w\) is the fundamental-wave winding factor, \(k_{\text{Cu}}\) is the winding fill factor, and \(J\) is the rms current density in the conductors.

The current density \(J\) can be solved from (21) if the electromagnetic torque is known for an operating point. The winding factor for the distributed three-phase air-gap winding shown in Fig. 1 is

\[
k_w = \frac{6}{\pi} \cos \frac{\pi}{6}.
\]

(22)

REFERENCES


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