Design and Comparison of Permanent Magnet Self-Bearing Linear-Rotary Actuators

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Abstract—Linear rotary actuators (LiRAs) are electric machines that can perform linear and rotary movements. They are used in many different applications, for example, for pick-and-place robots, in packaging or sorting lines, or as gearbox actuators. A linear-rotary movement can be obtained with various combinations of linear and rotary machines, whereas depending on the specifications of the underlying application the most suitable actuator arrangement has to be identified. In order to simplify the selection of the appropriate actuator configuration, this paper first gives an overview of possible realization concepts of linear-rotary actuators, which are also suitable to implement magnetic bearings (MB). Afterwards, fundamental scaling laws concerning achievable axial forces and torques of linear and rotary machines with interior and exterior rotor arrangement are derived, enabling a qualitative comparison in order to figure out the most suitable actuator concept. In this context, it is important that the derivation also considers the machine-internal heat flow and the heat dissipation to the ambient, which finally leads to a maximum current density depending on the selected topology. All findings are verified by finite element method simulations. In order to show the applicability of the derived scaling laws, a design example is discussed.

Index Terms—Current density, axial force, torque, linear-rotary actuators, tubular actuators, magnetic bearings, self-bearing.

I. INTRODUCTION

Linear-Rotary Actuators (LiRAs) are used in many different industries and application areas, for example, in electronics and semiconductor manufacturing industries in pick-and-place robots [1], [2], or in industries such as aerospace or automotive [3]–[5], food or pharmaceutical. Since LiRAs are used in so many different application areas, they have to meet different torque and/or force requirements, while also a given axial stroke has to be achieved. Consequently, choosing the most suitable actuator for given application specifications is not an easy task. Therefore, in this paper, torque and force scaling laws that give a quick and clear performance overview of different actuator arrangements are derived and verified with finite element method (FEM) simulations. Compared to scaling law derivations already done in literature [6], also thermal aspects are considered, which show to have a significant influence onto the optimal actuator geometry.

This paper focuses on permanent magnet (PM) LiRAs, as in general they have the highest power densities [7], but the derived scaling laws can also be applied to reluctance, flux switching or induction machines [2]. Besides the machine type, LiRAs can be realized in many different combinations of individual actuators, whereby also the coupling of the machines can be versatile, e.g. a parallel or series mechanical coupling, a magnetic coupling (e.g. checkerboard actuator [8]) or a double stator configuration [9], [10] can be used. However, as the LiRA with parallel mechanical coupling has a mechanical connection of the linear and rotary actuators [7], with independent rotors (also called ‘movers’ or ‘sliders’), it is not further considered in this work, since it would result in a bulkier and less robust design with lower acceleration performances due to higher moving mass and moment of inertia.

Another important aspect in LiRAs are the bearings. Most of the LiRAs use mechanical bearings, either ball or slider bearings. Besides their high stiffness and simplicity, both of those feature drawbacks, such as the need for lubrication and the particle generation. This is mainly a problem in applications where a high purity is required, e.g. in clean room applications. As an alternative, air bearings could be used, but they require a pressurized air supply and the operation in low pressure environments is prohibited. Accordingly, the mentioned issues can only be solved by magnetic bearings (MBs), which are gaining more and more attention in tubular linear and linear-rotary actuators [11]–[13].

This paper first summarizes possible options to realize a LiRA with MBs, and afterwards provides initial design considerations in terms of general scaling laws of electric machines that would help a potential designer to chose a topology suitable to the desired application. In contrast to the existing literature, the derived scaling laws also consider the machine-internal heat flow and the heat transfer to the ambient. Furthermore, the general scaling laws are applicable to any kind of electric machine and are also verified by FEM simulation.

II. ACTUATOR TOPOLOGY CONCEPTS

Depending on the application specifications, the LiRA with magnetic bearings (MB) can be built with different combinations of linear (L) and rotary (R) machines, as shown in Fig. 1. The considered actuators are divided into two groups, the first that features only an axial stator displacement of the different machines (i.e. linear, rotary or magnetic bearing) with either all interior rotor (cf. Fig. 2(a)) or all exterior rotor arrangements (cf. Fig. 2(b)), and the second group featuring a combined stator arrangement, i.e. a double stator LiRA (cf. Fig. 2(c)).

As can be noticed, for all possible LiRA arrangements always two independent magnetic bearings (MB) on each axial end are
required, such that rotor tilting can be controlled. Furthermore, depending on the degree of integration, the magnetic bearing can be either realized as an independent machine with separate stator (cf. LiRA-1 in Fig. 1) or can be integrated either into the rotary machine, i.e. a self-bearing rotary machine (MB+R) [15], or into the linear (L) machine (cf. LiRA-2 and LiRA-3 in Fig. 1), while a full integration of all three machines into a single machine is also possible (cf. LiRA-4 in Fig. 1). The realization options of these machines are shown in Fig. 3, where in the first row the different rotor’s permanent magnet arrangements and in the second row the corresponding stator’s winding configurations are given. As can be noticed, a rotary machine realized with a R-Rotor and R-Stator can also perform self-bearing (MB+R), while a linear machine (L) with L-Rotor and L-Stator doesn’t feature magnetic levitation. Hence, the integration of the magnetic bearing into a rotary machine (MB+R) is easier to realize compared to the integration of the MB into a linear machine (MB+L). In order to achieve self-bearing and linear movement in one machine, the L-Rotor must be combined with a CB-Rotor (‘Checkerboard-Rotor’, cf. [8]), as done in [13]. A further option to realize either a self-bearing rotary machine (MB+R) or a linear machine (L), is to use a S-Rotor (‘Square-Magnet-Rotor’, cf. [10]) with either a R-Stator or an L-Stator. Finally, to fully integrate all features into a single machine (MB+R+L), a CB-Rotor with the CB-Stator is needed. It should be noted that the same integration concepts (except the full integration) can also be applied to the double stator LiRA (cf. LiRA-5 and LiRA-6 in Fig. 1) and that the functionalities of the inner and outer stator can also be exchanged, i.e. the linear machine (L) would then be the outer actuator and the magnetic bearing (MB) together with the rotary machine (R) would be the inner actuator. Another aspect in LiRAs is the maximum axial stroke \( z_{\text{stroke}} \) that can be achieved with the selected machine arrangement. First of all, it has to be considered that the rotor or mover should be longer than the total stator length for at least \( z_{\text{stroke}} \), such that a constant interaction between the stator and the rotor is obtained. Furthermore, it has to be considered that depending on the selected stator and rotor arrangement, a certain distance \( \Delta z \) between the different stators is needed, which in case of an independent linear (L) or rotary (R) machine would have to be \( \Delta z = z_{\text{stroke}} \) (cf. LiRA-1 to LiRA-3 in Fig. 1), while for a fully integrated checkerboard machine or a double-stator machine no distance between the stators is needed, i.e. \( \Delta z = 0 \) (cf. LiRA-4 to LiRA-6 in Fig. 1). Indeed, LiRAs with an S- or CB-Rotor can be realized with \( \Delta z = 0 \), however, as will be shown in the following section, they also feature lower torque and force densities due to the inherently lower flux linkage of the S-Rotor [10] or the larger end windings of the CB-Rotor [16], and therefore finally result in a larger machine volume to achieve the same force and torque performances. For sake of completeness, the LiRA assemblies from Fig. 1 are listed in Tab. I for the different rotor and stator realizations given in Fig. 3 and it is shown whether a distance \( \Delta z \) between the stators is needed or not.

### III. Scaling Laws

In this section, the scaling laws for the achievable torque and the thrust force of the Interior Rotor and Exterior Rotor actuator arrangements are derived. In contrast to other literature [6], the current density amplitude \( J \) is calculated from the thermal (cooling) considerations, which have a significant influence on the achievable torques and forces. The scaling laws are verified with FEM simulations for the stator and rotor realizations shown in Fig. 3. The considered parameters are given in Tab. I.

Furthermore, it is assumed that the thickness of the rotor, the stator back iron, the permanent magnet, and the air gap are identical for all machines and compared to the outer dimensions, i.e. the inner and outer radii \( r \) and \( R \) as well as the length \( L \) of stator, are negligible. Moreover, also the air gap flux density \( B_{\text{ag}} \) is fixed to a constant value for all machines.

In [6], the current density amplitude \( J \) is assumed to be constant. This work extends the approach and \( J \) is calculated from the thermal (cooling) considerations, which, as shown later, significantly influence the achievable torque and force.

#### A. Interior Rotor

1) Torque Scaling Law: According to the fundamental expression for 3-phase electric machines, the torque magnitude \( T_{\text{int}} \) is proportional to the product of the flux linkage \( \Psi \) and the current amplitude \( I_{\text{int}} \) of the symmetric 3-phase winding system, i.e. \( T_{\text{int}} \sim \Psi I_{\text{int}} \). The flux linkage \( \Psi \) is the total flux linked with the \( N \) turns of the stator winding. Therefore, the flux \( \Phi \) that penetrates the stator from the air gap is \( N \) times smaller,
i.e. $\dot{\Psi} = N\dot{\Phi}$, and consequently the torque is proportional to $T_{\text{int}} \propto \dot{\Phi} \cdot N^2 T_{\text{int}}$. Furthermore, $\dot{\Phi}$ is proportional to the flux density in the air gap $B_{ag}$ and the air gap area $A_{ag}$, while $N^2 T_{\text{int}}$ represents the magnetomotive force, which can be written as the product of the current density amplitude $J_{\text{int}}$ and the winding area $A_{w}$, i.e. $N^2 T_{\text{int}} = J_{\text{int}} A_{w}$. Finally, the torque is proportional to $T_{\text{int}} \propto A_{ag} A_{w} B_{ag} J_{\text{int}}$.

The air gap and winding areas, $A_{ag}$ and $A_{w}$, can be further expressed by the geometrical parameters $R$ and $r$ (the outer and inner radii of the winding volume) shown in Fig. 4(a). For the air gap area the expression $A_{ag} \sim r L$ is applied, where $L$ equals the assumed stator length, and for the winding area the expression $A_{w} \sim (R^2 - r^2)$ is used. Since in the conducted analysis, the air gap flux density $B_{ag}$ is assumed to be constant, the torque can be scaled as $T_{\text{int}} \sim r (R^2 - r^2) L \cdot J_{\text{int}}$ which corresponds with the scaling law deduced in [6]. If a relative parameter $x_{r} = r/R$ is introduced, the torque is obtained as

$$T_{\text{int}} = K_T \cdot R^3 L \cdot x_{r} (1 - x_{r}^2) \cdot J_{\text{int}},$$

where $K_T$ is an absolute torque constant that is given in Tab. III for the analyzed LiRAs.

As already mentioned, in contrast to the constant current density amplitude $J_{\text{int}}$ assumed in [6], in the following a loss-dependent current density $J_{\text{int}} = J_{\text{int}}(P_{\text{cu}})$ is considered, which is given by the maximum allowed copper losses $P_{\text{cu}}$ in the stator windings. Based on the $P_{\text{cu}} = 1/2 R_{\text{cu}}^2 J_{\text{int}}^2$, the current density

**TABLE II:** Parameters used in FEM simulations.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Value/Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometrical</strong></td>
<td></td>
</tr>
<tr>
<td>Length $(L)$</td>
<td>100 mm</td>
</tr>
<tr>
<td>Outer Radius $(R)$</td>
<td>100 mm</td>
</tr>
<tr>
<td>Rotor Back Iron Thickness</td>
<td>2 mm</td>
</tr>
<tr>
<td>PM Thickness</td>
<td>2 mm</td>
</tr>
<tr>
<td>Number of Rotor Poles for Rotation</td>
<td>16(8)</td>
</tr>
<tr>
<td>Number of Rotor Poles for Linear Motion</td>
<td>16(8)</td>
</tr>
<tr>
<td>Number of Stator Teeth for Rotation</td>
<td>6</td>
</tr>
<tr>
<td>Number of Stator Teeth for Linear Motion</td>
<td>12</td>
</tr>
<tr>
<td>Total LiRA Volume $(V)$</td>
<td>$\pi R^2 L$</td>
</tr>
<tr>
<td>Stator Volume $(V_{\text{stator}})$</td>
<td>$\pi (R^2 - r^2) L$</td>
</tr>
<tr>
<td>Relating Winding Radial Size $(x_{r})$</td>
<td>$r/R$</td>
</tr>
<tr>
<td><strong>Magnetic / Electrical</strong></td>
<td></td>
</tr>
<tr>
<td>PM Remanent Flux Density</td>
<td>1.3 T</td>
</tr>
<tr>
<td>Rotor/Stator Core Relative Permeability</td>
<td>10,000</td>
</tr>
<tr>
<td>Copper Specific Electric Resistance at $T_w$ $(\rho_{\text{cu}})$</td>
<td>$2.36 \times 10^{-8} \Omega \cdot m$</td>
</tr>
<tr>
<td>Relative Copper Volume $(k_{\text{cu}} = V_{\text{cu}}/V_{\text{stator}})$</td>
<td>0.36</td>
</tr>
<tr>
<td>Current Density Constant $(K_J)$</td>
<td>$2 \sqrt{\Delta T/\rho_{\text{cu}}}$</td>
</tr>
<tr>
<td><strong>Thermal</strong></td>
<td></td>
</tr>
<tr>
<td>Winding Temperature $(T_w)$</td>
<td>120 °C</td>
</tr>
<tr>
<td>Ambient Temperature $(T_{\text{amb}})$</td>
<td>40 °C</td>
</tr>
<tr>
<td>Temperature Difference $(\Delta T)$</td>
<td>$T_w - T_{\text{amb}}$</td>
</tr>
<tr>
<td>Heat Transfer Coefficient $(h)$</td>
<td>10 W K$^{-1}$ m$^{-2}$</td>
</tr>
<tr>
<td>Winding Thermal Conductivity $(\lambda_{\text{cu}})$</td>
<td>2 W K$^{-1}$ m$^{-2}$</td>
</tr>
<tr>
<td>Iron Core Thermal Conductivity $(\lambda_{\text{fe}})$</td>
<td>22 W K$^{-1}$ m$^{-2}$</td>
</tr>
</tbody>
</table>

*Pole number values for S-[Rotor] are in brackets.
**Measured value, see [17].

...can be expressed as $J_{\text{int}} = \sqrt{(2P_{\text{cu}})/\rho_{\text{cu}} V_{\text{cu}}}$, where $\rho_{\text{cu}}$ is the specific resistance of copper and $V_{\text{cu}}$ the copper volume of the stator, which is given as $V_{\text{cu}} = k_{\text{cu}} V_{\text{stator}}$ (cf. Tab. II).

The allowed copper losses $P_{\text{cu}}$ are deduced from the actuator’s thermal properties, whereby the two heat transfer modes are considered: (1) radial heat flow through the windings by thermal conduction, modelled by the thermal resistance $R_{\text{th}}^h$, and (2) radial heat convection on the outer surface of the actuator to the environment, modelled by the thermal resistance $R_{\text{out}}$, which assumes a certain loss per surface area. The two thermal resistances can be obtained as

$$R_{\text{th}}^h = \frac{1}{h} \frac{\ln(R/r)}{2\pi L}, \quad R_{\text{out}} = \frac{1}{h} \frac{1}{2\pi RL},$$

where $\lambda_w$ is the specific thermal conductivity of the winding and $h$ is the heat transfer coefficient from the actuator’s outer surface to the environment (cf. Tab. II). The assumed thermal model is shown in Fig. 4(b). The allowed copper losses are obtained as $P_{\text{cu}} = \Delta T/(R_{\text{th}}^h + R_{\text{out}})$.

The copper volume can be calculated as $V_{\text{cu}} = k_{\text{cu}} \cdot \pi (R^2 - r^2) L$, where $k_{\text{cu}}$ is considering the amount of copper volume relative to the total stator volume. Assuming a winding fill factor equal to 0.6 and winding volume to stator volume ratio of 0.6, i.e. 60% while 40% is iron, $k_{\text{cu}}$ is calculated as $k_{\text{cu}} = 0.6 \cdot 0.6 = 0.36$ (cf. Tab. II).

Accordingly, the current density amplitude is calculated as

$$J_{\text{int}} = K_J \cdot \frac{1}{R} \sqrt{1 - x_{r}^2} \frac{1}{\lambda_w} \frac{1}{\sqrt{\ln(1/x_{r}) + 1/\lambda_w}}.$$ (3)

where $K_J$ is given in Tab. II.

The loss- and geometry-dependent current density $J_{\text{int}}$ can now be used in (3), in order to obtain the expression for the loss- and geometry-dependent torque $T_{\text{int}}$ of the interior rotor actuator. Another important quantity is the torque density $t_{\text{int}} = T_{\text{int}}/V$, which equals the torque $T_{\text{int}}$ divided by the total rotary actuator volume $V$ (cf. Tab. II) and results in the following expression

$$t_{\text{int}} = \frac{K_J K_J}{\pi} \cdot x_{r} \sqrt{1 - x_{r}^2} \frac{1}{\sqrt{\ln(1/x_{r}) + 1/\lambda_w}}.$$ (4)

The first factor is constant, while the second term only depends on the relative quantity $x_{r}$. The last factor, which comes from the thermal considerations, depends on both, the relative parameter $x_{r}$ and the absolute parameter $R$. Additionally, the last factor depends on the thermal parameters $\lambda_w$ and $h$. In order to examine the influence of these two thermal parameters, the extreme cases when $\lambda_w \rightarrow \infty$ or $h \rightarrow \infty$ are analyzed. Both cases can be physically interpreted and are shown in Fig. 5(a).

If $\lambda_w \rightarrow \infty$, then $R_{\text{th}}^h \rightarrow 0$, which means that the temperature drop inside the windings can be neglected. This can be related to the scenario in which the heat transfer coefficient $h$ is low (e.g. natural air cooling), i.e. heat transfer to the ambient is so low such that the temperature drop inside the winding becomes negligible. In this scenario, the torque density $t_{\text{int}}$ depends on the absolute value of the outer radius as $t_{\text{int}} \sim \sqrt{R}$ and its maximum is achieved for $x_{r} = 0.707$ (maximum of the function $x_{r} \sqrt{1 - x_{r}^2}$, cf. red curve in Fig. 5(a)).

If $h \rightarrow \infty$, then $R_{\text{out}} \rightarrow 0$, which means the case temperature of the actuator is fixed. This corresponds to the scenario where the heat transfer coefficient $h$ would be very high (e.g. water cooling), such that the main temperature drop occurs inside the machine. This is represented with the green curve in Fig. 5(a), which is a monotonically increasing function, since in case the windings get thinner (increasing $x_{r}$), the thermal resistance $R_{\text{th}}^h$ of the winding in radial direction is decreasing. Consequently,
more copper losses can be dissipated and a higher torque can be generated.

The curve that considers both, inner and outer thermal resistances, is always below the curves of the discussed scenarios (Fig. 5(a)), as it is limited by both thermal resistances. This curve is also verified with FEM simulations as shown in Fig. 5(b).

2) Thrust Force Scaling Law: Similar to the torque, the thrust (axial or drive) force $F_{\text{int}}$ is proportional to the flux linkage and the 3-phase current amplitude, $F_{\text{int}} \sim \Psi f_{\text{int}}$ and therefore is proportional to $F_{\text{int}} \sim A_{\text{ag}} A_w B_{\text{ag}} f_{\text{int}}$. The air gap and winding areas, $A_{\text{ag}}$ and $A_w$, can be deduced by using the geometrical parameters $R$ and $r$ from Fig. 4(a). Similar to the derivation from Sec. III-A1, $A_{\text{ag}} \sim rL$, while in the case of a linear actuator, the winding area only depends linearly on the radius and further is independent of the length $L$, i.e., $A_w \sim (R^2 - r^2)$. Assuming the air gap flux density $B_{\text{ag}}$ to be constant, the force is proportional to $F_{\text{int}} \sim r(R^2 - r^2) L \cdot J_{\text{int}}$, and by using the same relative parameter $x_r$, can be written as

$$F_{\text{int}} = K_F \cdot R^3 L \cdot x_r (1 - x_r) \cdot J_{\text{int}},$$

where $K_F$ is an absolute axial force constant that is given in Tab. III for the analyzed LiRAs.

The cooling properties are assumed to be the same as in the case of the rotary actuator, thus the current density $J_{\text{int}}$ is also given with (3) and can be inserted into (5). Similar to the torque density, the force density $f_{\text{int}}$ can be derived by dividing the force $F_{\text{int}}$ by the total linear actuator volume $V$ (cf. Tab. II), which results in

$$f_{\text{int}} = K_F K_3 \frac{1}{\pi} \cdot \frac{1}{R} \cdot x_r \sqrt{\frac{1 - x_r}{1 + x_r}} \cdot \frac{1}{\sqrt{\ln(1/x_r)} + 1/hR}.$$  \hfill (6)

Compared to the torque density in (4), the force density has a factor $1/R$, which means that the force density is increasing with a decreasing actuator’s outer radius. Hence, linear actuators (motors) are typically build with rather high length over radius ($L/R$) ratios.

The last factor in (6), which considers the thermal properties of the machine, is the same as the one in (4), therefore a discussion similar to Sec. III-A1 is conducted here.

Fig. 7: Tubular linear rotary actuator with Exterior Rotor with (a) radial and (b) axial cross sections. Based on the axial mounting of the interior stator, only axial heat flow is assumed and denoted with the red arrows. (c) Corresponding lumped parameter steady-state thermal model.

If $\lambda_w \to \infty$, the force density depends on the absolute outer radius as $f_{\text{int}} \sim 1/\sqrt{R}$. The influence of the relative parameter $x_r$ is reduced to $f_{\text{int}} \sim x_r \sqrt{(1 - x_r) / (1 + x_r)}$, which is shown in Fig. 6(a) with the red curve $f_{\text{int}}(\lambda \to \infty, h)$. In this scenario, the maximum force density is achieved for $x_r = 0.618$.

If $h \to \infty$, the force density depends on the absolute outer radius as $f_{\text{int}} \sim 1/R$, and in addition is again monotonically increasing with the relative parameter $x_r$. Again, the curve that considers both heat transfer coefficients $\lambda_w$ and $h$ is always smaller than the curves where only one of these parameters is considered. The verification with FEM simulations is shown in Fig. 6(b).

B. Exterior Rotor

1) Torque Scaling Law: In analogy to the actuator with interior rotor, the torque $T_{\text{ext}}$ of the actuator with exterior rotor is proportional to $T_{\text{ext}} \sim A_{\text{ag}} A_w B_{\text{ag}} J_{\text{ext}}$. Based on the geometric dimensions given in Fig. 7, the air gap area can be expressed by $A_{\text{ag}} \sim RL$, and the winding area by $A_w \sim (R^2 - r^2)$. Moreover, with a constant air gap flux density $B_{\text{ag}}$, the torque is calculated as $T_{\text{ext}} \sim R(R^2 - r^2) L \cdot J_{\text{ext}}$. Finally, using the relative parameter $x_r = r/R$, the torque becomes

$$T_{\text{ext}} = K_T \cdot R^3 L \cdot (1 - x_r^2) \cdot J_{\text{ext}},$$

where $K_T$ is again the absolute torque constant given in Tab. III.

For the exterior rotor actuator, i.e. interior stator actuator, the stator can only be mechanically fixed at one of the axial ends, therefore leading to an axial heat flow in the actuator (cf. Fig. 7(b)). The end with the mechanical fixation is assumed to have a heatsink with an area equal to $\pi R^2$, and a heat transfer coefficient $h$, which results in an outer thermal resistance $R_{\text{th}}$ (cf. Fig. 7(c)). Accordingly, due to the axial heat flow, the hot spot temperature is on the opposite axial end, with the temperature $T_{\text{th}}$ (cf. Fig. 7(b)). It is assumed that the axial heat flow occurs only in the stator back iron (cylinder with the radius $r$), while it is neglected through the winding volume, since the thermal conductivity in the winding is mainly inhibited by the poor conductance of the wire isolation and potting material ($\lambda_w / \lambda_{\text{th}} \sim 10$). Furthermore, as the copper losses $P_c$ are distributed in the winding volume, the heat generation is also spatially distributed along the stator, resulting in an inner thermal resistance $R_{\text{th}}$, to be half of the total back iron’s thermal resistance $R_{\text{th}}$, i.e., $R_{\text{th}} = R_{\text{th}}/2$. Accordingly, the thermal resistances for the tubular actuator with the exterior rotor can be calculated as

$$R_{\text{th}} = \frac{1}{2\lambda_{\text{th}} \pi R^2}, \quad R_{\text{th}} = \frac{1}{h \pi R^2},$$

where $\lambda_{\text{th}}$ is the thermal conductivity of iron given in Tab. II. Applying the same considerations as in Sec. III-A1, the allowed current density is obtained as

$$J_{\text{ext}} = K_3 \frac{1}{\sqrt{L}} \cdot \frac{1}{\sqrt{1 - x_r^2}} \cdot \frac{1}{\sqrt{\lambda_{\text{th}} x_r^2 + h}}.$$
As can be noticed, the torque density $f_{\text{ext}}$ depends on the ratio of the absolute outer dimensions $R$ and $L$, which means that making the actuator longer reduces the torque density due to the worse axial heat flow. Similarly to Sec. III-A1, the two extreme scenarios $h \to \infty$ or $\lambda R \to \infty$ can be analyzed (cf. Fig. 8(b)).

Again, the scaling law considering both thermal parameters is verified with FEM simulations as shown in Fig. 8(b).

2) Thrust Force Scaling Law: In analogy to the derivation done for the interior rotor, the thrust force $F_{\text{ext}}$ is given as $F_{\text{ext}} \sim A_{\text{ag}} A_{\text{A}} B_{\text{ag}} f_{\text{ext}}$. The air gap and the winding areas are again proportional to $A_{\text{ag}} \sim R L$ and $A_{\text{w}} \sim (R - r)$ and with the assumption of a constant air gap flux density $B_{\text{ag}}$, the force is proportional to $F_{\text{ext}} \sim R (R - r) L \cdot J_{\text{ext}}$. By using $x_r = r/R$, the previous expression can be written as

$$F_{\text{ext}} = K_F \cdot R^2 L \cdot (1 - x_r) \cdot J_{\text{ext}},$$

(11)

where $K_F$ is the force constant and the current density is given with (9). Hence, the force density for the tubular actuator with exterior rotor is obtained as

$$f_{\text{ext}} = \frac{K_F K_J}{\pi} \cdot \frac{1}{L} \cdot \sqrt{\frac{1 - x_r}{1 + x_r}} \cdot \frac{1}{L} \cdot \sqrt{\frac{1}{\lambda R \cdot \pi^2 x_r^2} + \frac{2}{h}}$$

(12)

The force density $f_{\text{ext}}$ only depends on the absolute length $L$ and decays when the length $L$ of the actuator increases. This effect is pronounced due to the axial heat flow in the actuator. The influence of the thermal parameters $\lambda R$ and $h$ is analyzed and shown in Fig. 9(a), while the verification by FEM simulation is shown in Fig. 9(b).

In general, for the tubular actuator with exterior rotor and therefore internal axial heat flow, the consideration of the thermal aspects is very important, since they influence the actuator geometry significantly as shown in Fig. 8(a) and Fig. 9(a).

C. Scaling Law Constants

In this section, the absolute values of the scaling law constants $K_T$ and $K_F$ are given and briefly discussed. Tab. III summarizes the constants for the actuators with \{R,L\}-Stator and \{R,L\}-Rotor for the interior rotor and with \{R,L\}-Rotor and \{R,L\}-Stator for the exterior rotor. The actuator constants for the rest of the actuator arrangements from Fig. 3 will be analyzed in future work.

As intuitively expected, the actuator constants for the S-Rotor are around 2 times lower compared to the R-Rotor and L-Rotor. This is the consequence of the 2 times lower PM cross section area, and therefore around 2 times lower flux linkage. More detailed analysis of the S-Rotor and its application in high dynamic positioning systems is explained in [10].

IV. DESIGN EXAMPLE DISCUSSION

As an example, in this section, the two possible realization options of the LiRA-1 (MB, L, R, MB) with interior rotor are compared, i.e. where either a combination of an L- and R-Rotor or a S-Rotor is used (cf. Fig. 1 and Fig. 3). In a first step, the magnetic bearings are not considered, since on the one hand the MBs are not yet considered in the scaling laws, and on the other hand the scaling laws are also applicable to machines with conventional bearings. Furthermore, the design discussion is conducted for the dimensions also used for the FEM simulations as given in Tab. II. Thereby, the length $L$ equals the total length of the complete actuator, which means that the axial stroke ($\Delta z$), the rotary actuator length $L_R$ and the linear actuator length $L_L$ have to be accommodated in the total length $L$. As already discussed, for the combined LR-Rotor the distance $\Delta z$ between the linear and rotary machines must be at least as large as the specified maximum stroke $z_{\text{stroke}}$, while for the S-Rotor no distance between the machines is needed ($\Delta z = 0$). However, it also must be mentioned that with the S-Rotor lower torque and force constants are achieved (cf. Tab. III), and therefore the volumes of the rotary machine $V_R$ and linear machine volume $V_L$ are bigger in order to achieve the same absolute torques and forces. Hence, considering the volume between the machines defined by $\Delta z$ as additional actuator volume $V_z$, the question arises for which range of stroke $z_{\text{stroke}}$, which machine realization results in a smaller overall actuator volume $V_A$ if a given absolute torque $T$ and force $F$ must be achieved. The total actuator volume is actually defined as $V_A = V_R + V_L + V_z$, which, based on the assumption of a constant outer radius $R$ for all machines, corresponds to $L = L_R + L_L + \Delta z$. Accordingly, in case of the RL-Rotor with increasing $\Delta z$, the remaining length for $L_R$ and $L_L$ is reduced, which in consequence also leads to a reduction of the maximum achievable force and torque performance, while for the S-Rotor always the full actuator length $L$ can be shared between the two machines. Moreover, for both actuator realizations, the length distribution between $L_R$ and $L_L$ can be selected arbitrarily. E.g. in the extreme case, where $L_R = 0$ and $L_R = L - \Delta z$, the actuator achieves the maximum torque but no axial force is obtained, i.e. only a rotary machine. However, if now $L_R$ is decreased, also the maximum achievable torque decreases linearly, since $T = t \cdot V_R = t \cdot L_R$, and the axial force linearly increases, since $F = f \cdot V_L \sim f \cdot L_L$.

TABLE III: Scaling law constants determined by FEM simulations.

<table>
<thead>
<tr>
<th>Rotor</th>
<th>Stator</th>
<th>$K_T$ (N m A$^{-1}$ m$^{-3}$)</th>
<th>$K_F$ (N A$^{-1}$ m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior Rotor</td>
<td>R</td>
<td>0.83</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>0.35</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>-</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>-</td>
<td>0.48</td>
</tr>
<tr>
<td>Exterior Rotor</td>
<td>R</td>
<td>0.71</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>-</td>
<td>0.99</td>
</tr>
</tbody>
</table>
This behavior is visualized in Fig. 10(a) for different stroke lengths $z_{\text{stroke}}$. As can be clearly noticed, for the LR-Rotor, the achievable torque-force-ratio decreases with increasing $z_{\text{stroke}}$ and for a maximum stroke of $z_{\text{stroke}} = 100 \text{mm}$ neither an axial force nor a torque can be achieved. On the other hand, for the S-Rotor, a stroke-independent torque-force-ratio is obtained. In this case, the break even in performance is roughly found at the half of the total actuator length $L/2 = 50 \text{mm}$, which means that for axial strokes smaller than $L/2$ the LR-Rotor performs better, while for $z_{\text{stroke}} > L/2$ the S-Rotor should be used (cf. yellow shaded area in Fig. 10(b)). The factor $1/2$ actually comes from the ratio of the force and torque constants $K_{T}$ and $K_{F}$, which for the two rotor type roughly differs by this factor. Hence, $z_{\text{stroke}}$ of the break-even point can easily be estimated by writing $L_{R,LR} + L_{I,LR} + \Delta = L_{R,S} + L_{I,S} + \Delta$, where $K_{T,LR} \cdot L_{I,LR} = K_{T,S} \cdot L_{R,S}$ and $K_{F,LR} \cdot L_{I,LR} = K_{F,S} \cdot L_{R,S}$. This behavior is visualized in Fig. 10(b). The two normalized torques and force densities achieve for the specifications given in Tab. II in are shown. Accordingly, the LR-rotor would be realized with two different diameters. Here for the linear machine the optimum radius ratio is $x_{r} = 0.711$ and for the linear machine $x_{r} = 0.624$. For the S-Rotor a compromise between torque and force has to be made, which, for the given actuator dimensions is found at $x_{r} = 0.67$. As can be noticed, this sub-optimal radius ratio is hardly decreasing the achievable force and torque densities, however, for other actuator dimensions can be much larger, which means that the break-even point concerning achievable performance is shifted to even larger strokes $z_{\text{stroke}}$.

**CONCLUSIONS**

This paper gives an overview of possible realization concepts to build a linear-rotary actuator (LiRAs) with magnetic bearings (MB), i.e. a self-bearing electric machine that can realize coupled linear and rotary movements. In order to help the designer to easily compare different realization options and to simplify the selection of the appropriate actuator concept for a given application, general scaling laws concerning torque and forces considering also the heat flow inside and outside the actuator are deduced for interior and exterior rotor arrangements. All the findings are verified with FEM simulations. The scaling laws are also applicable to special actuators (checkboard or double-stator) as well as to standard rotary and/or linear actuators with conventional bearings, as was also done for a design example in this paper. The comparison of linear-rotary actuator realized with either separate linear and rotary machines or a combined linear-rotary machine showed, that the separate realization outperforms the combined actuator with respect to the total actuator volume as long as the linear stroke is smaller than half the length of the total actuator. Furthermore, depending on the outer dimensions given by the underlying application, this break-even point can be even shifted to larger stroke values, since for the separate realization both machines can be optimized independently, while for the combined actuator a compromise has to be made. The future work will focus onto further development of the scaling laws and the comparison of the further LiRA arrangements.

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