Abstract—This paper treats the possibility of transferring the modulation method developed for a pulse width modulated (PWM) converter with a dc voltage link (VLC) to a PWM converter with dc current link (CLC) based on the inverse current and voltage behavior of these two systems. The duality between VLC and CLC has been intuitively clear. This “duality” is treated here in a scientific approach. Especially, transferring of modulation methods from VLC to CLC and vice versa are beneficial for practical applications.

The nonplanar property of three-phase bridge circuits leads to a quasidual relation of the switching states of the two systems. The code conversion of the binary switching functions of a VLC into control signals of the power electronic devices of a CLC is given. The remaining degree of freedom of the correspondence is set by the requirement of minimum switching frequency of the converter.

Index Terms—dc current link PWM converter, dc voltage link PWM converter, duality relation, space vector modulation

I. INTRODUCTION

THREE-PHASE pulse width modulated (PWM) dc voltage link converters (VLC) and dc current link converters (CLC) have found wide areas of applications in industry within the last few years [1]–[3]. Such areas are PWM inverters (e.g., for drives), PWM rectifiers, static-var-compensators, etc. There exists a multitude of publications dealing with the theory of the two converter systems (VLC: [4]–[8], CLC: [9]–[13]).

Despite the fact that the inverse current/voltage behavior of VLC and CLC has been mentioned in different publications [14], [15], the comparison so far has been limited to a discussion of the effort connected with the practical realization or to general technical aspects of the two systems. Although the duality between VLC and CLC has been intuitively clear to most experts in power electronics, thus far, no thorough and scientific treatment of this duality has been presented in literature to our knowledge. This has been the motivation for the research described here. This research deals with the structural relation and the correspondence of the control methods of the two converters (see Sections II and III). As shown in Sections IV and V, the derived relations make it easily possible to convert the control methods known for VLC to methods for CLC and vice versa. The extremely involved calculations of modulation methods being optimal with respect to, e.g., harmonics then can be replaced by a simple conversion of the relevant pulse patterns if an equivalent modulation method is known for the other converter type.

Because more research results can be found in the literature dealing with pulse pattern optimization for VLC than for CLC, this paper only treats the transfer of control methods from VLC to CLC.

II. FUNDAMENTALS OF DUALITY RELATIONS

As already mentioned in Section I, the duality of the converters shown in Fig. 1 is mentioned, e.g., in [14] and [15], regarding the time behavior of voltages and currents on the ac and dc sides. Before a detailed analysis is performed, in this section, the most important basics of the duality of electrical networks shall be summarized briefly. Regarding a detailed treatment, see [16]–[20].

The necessary and sufficient condition for the existence of a dual network of a given primary circuit is the planar property of the graph of the primary circuit. This is given if the graph does not contain any of the Kuratowski graphs shown in Fig. 2 as a subgraph. Then, the graph can be drawn in a plane without an intersection of the branches.

As an example for constructing a dual network, we want to consider a single-phase dc voltage-link converter (Fig. 3). As in Fig. 1, we only treat the essential aspects; therefore, we consider the elementary converter and replace the peripheral devices by voltage and current sources.

As shown in Fig. 3 (right-hand side), the dual graph of the circuit is found by connecting the nodes 1...4. These nodes have a duality relation to the meshes of the primary circuit. Each branch of the dual graph intersects exactly one branch of the graph of the primary network. This gives a correspondence of the branches. As the primary graph, the dual graph also shows a bridge structure. The transfer of the orientations given in the primary graph to the dual graph is performed by counter-clockwise or clockwise rotation (as performed here) of the arrows, giving the positive directions of the currents and voltages until they overlap with the intersecting branches of the dual graph. This transfer of the orientations is especially important for power electronics due to the dependency of current and voltage characteristics on the direction of the power electronic devices.

To extend the dual graph to the dual network, we have to assign an electrical characteristic to each branch. This characteristic is derived from that of the corresponding branch of the primary network by exchanging voltage and current.

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Fig. 1. Structure of the power circuit of a three-phase VLC (left) and of a three-phase CLC (right). Elementary structures are shown, and peripheral circuit elements are replaced by voltage and current sources.

Fig. 2. Kuratowski graphs; basic elements of nonplanar graphs.

Fig. 3. Development of the network being dual to a simplified single-phase VLC (right: primary network—left: dual network).

This exchange can always be performed, even for an arbitrary nonlinear behavior of the primary network. Therefore, for power electronic devices, dual elements can also be assigned directly [18]-[20].

In general, the following duality relations are valid:

- **Topological relations:**
  - node $\leftrightarrow$ loop
  - parallel connection $\leftrightarrow$ series connection
  - delta connection $\leftrightarrow$ wye-connection

- **Physical quantities:**
  - voltage $\leftrightarrow$ current
  - voltage time area $\leftrightarrow$ current time area
  - (magnetic flux $\leftrightarrow$ electrical charge)

- **Power sources:**
  - voltage source $\leftrightarrow$ current source

- **Network (Circuit) elements:**
  - capacitor $\leftrightarrow$ inductor
  - resistor $\leftrightarrow$ resistor (conductance).

Regarding the duality relations of power electronic devices, we have the following:

The element that is dual to a diode again has properties of a diode. The dual exchange blocking voltage/forward current leads to a reversal of the valve action, however. The direction of the transistors due to the equal orientation of blocking voltage and forward current remains unchanged for the dual transition. However, one has to apply an inverted control signal because the turn-on state of the primary transistor corresponds to the turn-off state of the dual transistor (and vice versa).

The dual relation for a valve combination that is characteristic for VLC can be easily given (cf., Fig. 4) if one considers the rules given above. A bidirectional, unipolar device therefore is transformed into a bipolar, unidirectional device with inverted control signal. For transformation of further valve combinations, see, e.g., [19].

Accordingly, the control signals of the valves of a single-phase VLC can be applied after inversion directly, at least in theory, for the control of the dual single-phase CLC. The switching of the valves of the same bridge leg that does not overlap in the former case (break before make) is to be replaced by overlapping switching (make before brake) for practical applications, however.

III. DUAL NETWORK OF A THREE-PHASE DC VOLTAGE LINK PWM CONVERTER

After this short discussion of the basics of dual relationships, we want to proceed to the real problem, i.e., the derivation of the dual structure of the three-phase VLC. An immediate transformation of the entire circuit is not possible due to the nonplanarity of three-phase bridge circuits. The three-phase
VLC shows the topology of one of the two basic elements of nonplanar graphs (Figs. 1, 2; cf., numbering of the nodes). This means that no dual network can be given.

A scientifically exact derivation of the relationship between VLC and CLC due to the circuit structure is therefore possible only separately for each switching state of the system. As shown in Fig. 5, we have planar active subcircuits for the different switching states whose dual correspondences can be given immediately. There, the switching states of the VLC are denoted by the switching state vectors $[s_R \ s_S \ s_T]$, which in turn are assembled by the binary phase switching functions $s_R$, $s_S$, and $s_T$. When a phase is connected to the positive dc link voltage bus, the related switching function becomes $s_i = 1$. If connected to the negative bus, $s_i = 0$.

A minimum-effort combination of the dual partial systems leads to the overall structure that is also shown in Fig. 5. This is to be called a quasi-dual circuit of a three-phase VLC because a dual circuit does not exist a priori. As is immediately clear by a comparison of Figs. 1 and 5, the quasi-dual circuit shows the topology of a CLC. The realization of the circuit elements that are to be inserted after the determination of the overall structure again (cf. Section II for single-phase systems) has to be performed by a series connection of a transistor and of a diode or, generally, by using a bipolar, unidirectional power electronic device.

The structure of a three-phase VLC built up by three single-pole, double-throw switches, therefore, has its quasi-dual correspondence in the structure of a three-phase CLC formed by two single-pole, triple-throw switches. This representation limited to the basic operating principle of the two systems very clearly shows the only quasi-dual relationship of the two converters because the VLC has $2^3 = 8$ (six active and two freewheeling states); the CLC, on the other hand, has $3^2 = 9$ (six active and three freewheeling states) possible switching states.

IV. SPACE VECTOR REPRESENTATION AND SWITCHING STATES OF DC VOLTAGE LINK AND DC CURRENT LINK PWM CONVERTERS

In the following, the line-to-line voltages and phase currents (duality of $Y$ and $\Delta$ connections) formed for the different system switching states are described and analyzed using space vector calculus [21]-[23]. Based on this, the transfer of a modulation method from a VLC to a CLC is described.

This transfer of the modulation method is done by representing pulse patterns (optimized with respect to harmonics,
Fig. 6. Space vector diagrams of the line-to-line voltages of a VLC (left) and of the phase currents of a CLC (right).

For the definition of the space vector of the line-to-line voltages of the VLC, we have

$$V_2 = \frac{2}{3}(v_{RS} + a v_{ST} + a^2 v_{TR})$$

$$= \frac{2}{3} V_{DC} [(s_R - s_S) + a (s_S - s_T) + a^2 (s_T - s_R)] \quad (1)$$

with

$$a = \exp \left( j \frac{2\pi}{3} \right) \quad a^2 = \exp \left( -j \frac{2\pi}{3} \right). \quad (2)$$

For the space vector of the phase currents of the CLC, we have, correspondingly

$$i_2 = \frac{2}{3}(i_{RS} + ai_{ST} + a^2 i_{TR})$$

$$= \frac{2}{3} I_{DC} [(s_{R+} - s_{R-}) + a (s_{S+} - s_{S-}) + a^2 (s_{T+} - s_{T-})]. \quad (3)$$

This results in the voltage and current space vectors [26]-[28] given in Fig. 6 from which the phase quantities can be determined by the inverse transformation [21] via projection of the space vector quantities to the corresponding axes.

The switching states of the systems are denominated by indices: for VLC—via switching state vectors $[s_R \ s_S \ s_T]$ (cf. Section III) and for CLC—via switching state matrices $[s_{R+} \ s_{S+} \ s_{T+}]$ and $[s_{R-} \ s_{S-} \ s_{T-}]$ formed in an analog manner.

For the active (voltage and current forming) system switching states, we can give a direct correspondence defined by voltage and current space vectors of equal phase and of equal magnitude (normalized with respect to the relevant dc link quantity)

$$\frac{|v|}{V_{DC}} = \frac{|i|}{I_{DC}} = \frac{2}{\sqrt{3}} \quad (4)$$

(cf. Table I). As a comparison of the particular active partial structures shows (cf. Fig. 5), this correspondence is identical with that derived in Section III. The freewheeling states that do not result in the formation of line-to-line voltages or phase currents are all represented by the voltage or current zero vector. Therefore, with respect to the voltage or current generation, no difference exists between the freewheeling states of a converter system, and no direct relationship can be derived between the freewheeling states of VLC and the CLC. Therefore, in general, for a freewheeling state of the VLC, any of the freewheeling states of the CLC has to be guaranteed.

The basic function of the considered converter systems now consists of approximating a given reference space vector $v^*$ or
Fig. 7. Illustration of the switching state sequences of a VLC (right) and a CLC (left) by a state diagram.

Each new switching state is reached by switching of only one bridge leg. Each freewheeling state ([111] or [000]) is present, alternating at the beginning and at the end of each pulse half period. The switching state sequence (which changes for each \( \pi/3 \) interval of the phase of \( v^* \)) can be illustrated graphically by a state graph as in Fig. 7 (left). The part in Fig. 7 that is marked clearly designates the switching state sequence given previously.

When this state graph (giving the most general form of a modulation method) is transferred to the quasi-dual CLC system (Fig. 7, right), the active switching states on the inner side are to be related via Table I. The three possible freewheeling states \([1 0 0], [0 1 0], [0 0 1]\) are to be incorporated sensibly into the switching state sequence in such a way that the current generation is performed with a minimum number of switchings. As shown in Fig. 7, we can reach two freewheeling states by one switching, starting from each active switching state (e.g., \([1 0 0]\) and \([0 0 1]\)). Among these freewheeling states, only that has to be selected that also results in a minimum number of switchings for a change of the switching state sequence due to the entering of \( v^* \) into the next \( \pi/3 \) interval (cf., dash-dot lines in Fig. 7). This is of importance especially for low pulse frequencies. One of the two optimum variants is shown in Fig. 7.

V. CODE CONVERSION OF THE SWITCHING STATE VECTORS OF A dc VOLTAGE LINK PWM CONVERTER

The recoding of a switching state sequence (cf. Fig. 8) described in Section IV by relating the state graphs of VLC and CLC (cf. Fig. 7) can be performed simply by software or by a special hardware state sequencer. Fig. 9 shows the basic structure of a possible realization that can be implemented, e.g., by programmable logic (PAL, EPLD, LCA, etc.).

The switching state vector \( s_k \) of a VLC being present at the address inputs A0...A2 of the ROM is being recoded into the switching state matrix \( g_k \) of the quasi-dual CLC.
Fig. 9. State sequencer for conversion of the switching state vector $s_{k+1} = [s_{R+}, s_{R-}, s_{T+}, s_{T-}]$ of a VLC into the switching state matrix $s_k = [s_{R+}, s_{R-}, s_{T+}, s_{T-}]$ of a CLC.

As the switching-frequency-optimal state graph of a CLC (cf. Fig. 7) shows, the last and the last-but-one switching state are to be considered here. This is made possible by a two-stage latch-chain (latch 1 and latch 2), which is being triggered when a new switching state vector $s_{k+1}$ occurs. The triggering signal is formed by logic differentiation (i.e., evaluation of the difference of the input and output of latch 3; cf. Fig. 9) of signal $s_{k+1}$. For the sake of clarity, we have not presented in Fig. 9 the latches at the input and output of the state sequencer. These latches are operated with the high system clock frequency and would have to be provided for a practical realization.

VI. CONCLUSIONS

VLC and CLC show, as becomes immediately clear from the comparison of the circuit structure of the two systems, a close relationship. However, due to the nonplanar graphs of the systems, they do not show exactly dual behavior. By considering the circuit parts as being active for the different switching states, one can derive a quasi-dual relationship between VLC and CLC, which has been taken as basis of combining the theory of the control of the two systems in this paper.

Although, as pointed out, the duality between VLC and CLC might have been intuitively clear thus far, a scientific treatment as presented here is important for practical applications. These are, e.g., the transfer of modulation methods known for VLC to CLC systems. This might save considerable time and effort for designing new systems.

In general, one has to point out that the dual transformation "performed in segments" of a nonplanar power electronic circuit also leads to a certainly that is already known, but—in any case—technically meaningful circuit. This point seems to be of special interest due to the increasing importance of the dual transformation of known circuit topologies for the gain of new circuit variants.

REFERENCES

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