Outrunner Generator with Optimized Cogging Torque Pattern for an Electromechanical Energy Harvester

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Outrunner Generator with Optimized Cogging Torque Pattern for an Electromechanical Energy Harvester

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Abstract—In contrast to widely applied permanent magnet (PM) machine optimization for minimization of cogging torque, this paper describes a method for shaping the cogging torque of a PM outrunner machine towards a desired sinusoidal torque pattern. The underlying goal of this approach is to compensate the cogging torque of a kinetic energy harvester (KEH) with the optimized counter cogging torque of a generator connected to the same shaft. Therefore, the total cogging torque is highly reduced, and a self-starting electromechanical energy harvester, comprising KEH and outrunner generator is formed. The key degree of freedom for shaping the cogging torque is the sinusoidal modulation of the machine’s air gap. An algorithm based on the multi-dimensional Secant method, which is related to the multi-dimensional Newton-Raphson method, first evaluates the cogging torque of a given generator geometry with two-dimensional finite element method (2-D FEM) simulations and then iterates the geometry of the outrunner machine until the cogging torque target is achieved. Using the presented optimization approach, a generator design with the desired sinusoidal cogging torque pattern is obtained, achieving a total cogging torque reduction of the overall electromechanical energy harvesting system of 90%.

I. INTRODUCTION

The cogging torque (detent torque) in conventional permanent magnet (PM) machines occurs due to a reluctance variation along the stator perimeter introduced by the stator teeth in combination with the PMs. It is even present without stator excitation [1] and typically, a minimization of the cogging torque is targeted in the course of machine design. This topic is widely analyzed for DC machines [2], permanent magnet synchronous machines (PMSM) [3–11] and brushless DC (BLDC) machines [12–14] in recent publications and in textbooks [1, 15, 16]. However, for certain applications as described in [17] and for the kinetic energy harvester (KEH) discussed in this paper, a specially shaped, non-vanishing cogging torque is desired.

The electromechanical energy harvester introduced in [18] harvests electrical energy/power without mechanical contact, from the kinetic energy/power of a moving conductive secondary (MCS). The system overview is given in Fig. 2. It comprises a KEH and a generator, that converts the harvested
kinetic energy/power to electric energy/power. The prototype presented in [18] harvests from an aluminum MCS and utilizes an off-the-shelf generator. In order to broaden the range of possible applications, it is desired to also allow MCSs made of a ferromagnetic material, e.g. steel.

A. System start-up

Measurements show that the KEH based on [18] can harvest a mechanical power of $P_{\text{mech}} \approx 10 \text{ W}$ in the steady-state operation at an air gap $g_{\text{KEH}} = 10 \text{ mm}$, when the speed of the MCS made of C45E steel is $v_2 = 22 \text{ m/s}$. However, the use of a MCS made of steel leads to an effect, which is not present when an aluminum MCS [18] is used. Due to the inherent partial overlap of KEH and MCS (cf. Fig. 2), a magnetically favorable rotational position of the KEH appears at standstill. In other words, a KEH, optimized for high energy harvesting capability, exhibits a high cogging torque. The cogging torque of the KEH of [18], depicted in Fig. 5, is calculated as a function of angular position with the aid of 3-D finite element method (FEM) simulations and verified with a precision torque sensor mounted on the same shaft as the KEH (cf. Fig. 4).

Literature [19–22] proposes electrical compensation of torque ripple and cogging torque of electrical machines. For the energy harvesting system as depicted in Fig. 2, this would mean to drive the generator in motor mode for the system start-up in order to overcome the KEH cogging torque. Consequently, energy from an external energy storage would be required to start up the energy harvester. Moreover, a control would be required to identify whether energy harvesting is possible (i.e. to detect if the MCS is in motion) or not (MCS not in motion; system shut down). However, for applying the proposed electromechanical energy harvesting system in industry, a self-start-up capability is highly beneficial, leading to the key idea of this work. If the generator (cf. Fig. 2) has the same amount, but opposite sign of cogging torque as the KEH for each rotational point, the total cogging torque of KEH and generator vanishes,

$$T_{\text{cog,gen}}(\varphi) + T_{\text{cog,hary}}(\varphi) = 0 \Rightarrow T_{\text{cog,gen}}(\varphi) = -T_{\text{cog,hary}}(\varphi).$$

(1)

B. Electromechanical energy harvester with integrated outrunner generator

Since the KEH has a disk-shaped geometry, the generator is beneficially implemented as an outrunner machine. This leads to a highly compact design as depicted in Fig. 1 and Fig. 3. The axially magnetized KEH PMs and the radially magnetized generator PMs are mounted on a pot-shaped steel rotor. It is intended to use different PMs for KEH and generator as it conveniently allows to decouple both machine designs. The rotor forms a yoke for all placed PMs and is of solid steel as it experiences negligible flux variation during operation. The stator, on the other hand, is made of laminated electrical steel.

This paper proposes the cogging torque cancelation of an already introduced kinetic energy harvester [18] with the counter cogging torque of an optimized PM outrunner machine. Sec. II
II. COGGING TORQUE OF SLOTLESS STATOR

As depicted in Fig. 5, the cogging torque of the KEH and therefore the desired inverse cogging torque of the generator can be approximated well with a sinusoidal function, exhibiting a period of 90° mechanical rotation. Prior to the actual stator optimization procedure in Sec. III, the approach is derived with the cogging torque analysis of a slotless (and windingless) stator.

Specially shaped magnets as proposed in [5] and magnets with variable width as proposed in [4] are excluded from the consideration as this would lead to mechanical unbalance, increased mounting effort and increased PM costs. Hence, only radially magnetized sector magnets are considered for the generator. A specific stator shape, resulting from the modulation of the air gap, is selected as the most suitable degree of freedom for the cogging torque shaping. Since the stator is made of laminated electrical steel and the individual sheets are laser cut in a prototyping process and punched in mass production, the optimized stator shape has negligible additional production cost compared to a conventional outrunner machine of similar size.

The cogging torque, either obtained by FEM simulations or analytical derivations, can be expressed with a Fourier expansion

\[ T_{\text{cog}}(\varphi) = c_1 \sin(4\varphi) + d_1 \cos(4\varphi) + c_2 \sin(8\varphi) + d_2 \cos(8\varphi) \ldots , \]  

where \( \varphi \) is the angle of rotational position. As it is desired (cf. (1)) to obtain a torque, shaped as the KEH torque in Fig. 5, but opposite in sign, all components in (2) except for \( c_1 \) should vanish.

A slotless stator geometry in combination with a sinusoidal PM magnetization that leads to the desired sinusoidal cogging torque is depicted in Fig. 6. The air gap modulation \( g(\theta) \) and the PM’s remanent flux density \( B_{\text{rem},r}(\theta, \varphi) \) are of the form

\[ g(\theta) = g_{\text{nom}} \cdot (1 + a_1 \cos(4\theta)) \]  
\[ B_{\text{rem},r}(\theta, \varphi) = B_{\text{rem}} \cos(2(\theta - \varphi)) , \]

where \( \theta \) is the geometric angle in the stator coordinate system and \( B_{\text{rem}} = 1.4 \text{T} \) for this analysis.

In a similar consideration as in [23], by applying Ampere’s law

\[ \oint_S \frac{\vec{B}}{\mu} \cdot d\vec{s} = \int_A \vec{j} \cdot d\vec{A} , \]

on any integration path (one is indicated in Fig. 6) including stator, air gap, PMs and the rotor and with assuming \( \mu_{\text{Fe}} \to \infty \), \( \mu_{\text{PM}} = \mu_0 \), the flux density in the air gap in radial direction...
III. COGGING TORQUE PATTERN OPTIMIZATION

The optimization is initialized based on the findings obtained from the slotless arrangement, depicted in Fig. 9 (a) and with simulation parameters according to Table I. Clearly, slots (with the angular width of the stator teeth $\alpha_{FE}$) are introduced for placing the stator coils. Additionally, the sinusoidal PM remanent flux density in Sec. II is replaced by PMs with constant PM remanent flux density $B_{rem, \varphi} = B_{rem} = 1.4$ T, where an angular gap between the PMs $\delta_{PM, gap}$ is introduced and part of the optimization.

A. Optimization Algorithm

A flowchart depicting the optimization algorithm is given in Fig. 8. After initializing the optimization with selected
TABLE I: Parameters used for the 2-D FEM simulations of the generator.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator bore diameter</td>
<td>(d_1)</td>
<td>67 mm</td>
</tr>
<tr>
<td>PM height</td>
<td>(h_{PM})</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>2-D rotor diameter(a)</td>
<td>(d_{rot, Ae})</td>
<td>120 mm</td>
</tr>
<tr>
<td>Actual rotor diameter</td>
<td>(d_{rot})</td>
<td>75 mm</td>
</tr>
<tr>
<td>Stator material</td>
<td></td>
<td>M235-35A</td>
</tr>
<tr>
<td>Rotor rel. permeability</td>
<td>(\mu_{rot, rel})</td>
<td>700</td>
</tr>
<tr>
<td>Stator width</td>
<td>(l_y)</td>
<td>10 mm</td>
</tr>
<tr>
<td>Number of pole pairs</td>
<td>(p)</td>
<td>2</td>
</tr>
<tr>
<td>Number of phases</td>
<td>(m)</td>
<td>3</td>
</tr>
<tr>
<td>Nominal air gap</td>
<td>(g_{nom})</td>
<td>2 mm</td>
</tr>
<tr>
<td>PM remanent flux density</td>
<td>(B_{rem})</td>
<td>1.4 T</td>
</tr>
<tr>
<td>PM rel. permeability</td>
<td>(\mu_{PM, rel})</td>
<td>1.05</td>
</tr>
<tr>
<td>Stator tooth width</td>
<td>(w_{FE})</td>
<td>10 mm</td>
</tr>
<tr>
<td>Coil cross section</td>
<td>(A_{coil})</td>
<td>143 mm(^3)</td>
</tr>
</tbody>
</table>

\(a\) Used to represent the actually pot-shaped rotor flux path in 2-D simulation.

geometry parameters, the current configuration is evaluated by calculating the cogging torque as function of the angle of rotational position \(\varphi\) with 2-D FEM simulations. Then, the utilized algorithm iterates the geometry until the cogging torque target is reached. Due to the numerical nature of the simulation, the previously introduced condition for completely vanishing total cogging torque (1) cannot be reached. Therefore, a torque residual

\[ T_{cog, rem} = T_{cog, harv} - T_{cog, gen} \] (11)

is computed and the iteration loop is terminated when the relative residual is sufficiently small \(T_{cog, rem}/T_{cog, harv} < \delta\). In the following optimization procedure, \(\delta = 0.1\) is selected as it leads to a sufficient reduction of total cogging torque of 90%.

\(T_{cog, harv}\) and \(T_{cog, gen}\) are functions of the continuous variable \(\varphi\) and therefore intrinsically difficult to handle in an optimization. A decomposition of the cogging torque according to (10) is performed and a vector of cogging torque harmonic components

\[ \mathbf{c} = \begin{bmatrix} c_1 & d_1 & \ldots & c_m & d_m \end{bmatrix}^T \] (12)

is obtained. A similar decomposition can be performed for \(T_{cog, harv}\) and (11) can be rewritten as

\[ \mathbf{c}_{rem} = \mathbf{c}_{harv} - \mathbf{c}_{gen} \]. (13)

On the other hand, it is also convenient to represent the geometry parameters in a vector

\[ \mathbf{a} = \begin{bmatrix} \alpha_{FE} & \alpha_{PM gap} & b_1 & \ldots & a_n & b_n \end{bmatrix}^T \]. (14)

The modulation index \(a_1\) (cf. (3)) is not updated during the optimization and is used as an external geometry parameter.

It is set \(a_1 = 0.5\) in the presented results.

The update of geometry parameters is a multi-dimensional Secant Method, which is related to the multi-dimensional Newton-Raphson Method [24], but utilizes the secant as an approximation for the Jacobian matrix,

\[ \frac{\partial \mathbf{c}}{\partial \mathbf{a}_j} \approx \mathbf{J} = \begin{bmatrix} \frac{\Delta c_1}{\Delta a_{FE}} & \cdots & \frac{\Delta c_1}{\Delta a_{PM}} \\ \vdots & \ddots & \vdots \\ \frac{\Delta c_m}{\Delta a_{FE}} & \cdots & \frac{\Delta c_m}{\Delta a_{PM}} \end{bmatrix} \], (15)

for finding the root (zero) of the error function (\(\mathbf{c}_{rem}\) here) [25]. For obtaining e.g. the first column in \(\mathbf{J}\), the parameter \(\alpha_{FE}\) is perturbed and a simulation is conducted. The difference in terms of cogging torque decomposition \(\mathbf{c}_{gen}\) gives the values for the first column. This procedure is conducted for all elements in geometry parameter \(\mathbf{a}\), such that \(\mathbf{J}\) can be filled.

Then the geometry parameter \(a\) is updated such that the torque residual \(\mathbf{c}_{rem}\) is reduced,

\[ a_{i+1} = a_i - d_{damping} \mathbf{J}^+ \mathbf{c}_{rem} \]. (16)

The pseudoinverse of the approximated Jacobian matrix,

\[ \mathbf{J}^+ := \left( \mathbf{J}^T \mathbf{J} \right)^{-1} \mathbf{J}^T \],

is used for the update step since the existence of \(\mathbf{J}^{-1}\) cannot be guaranteed. Due to the strong nonlinearity of the optimization problem, a damping factor \(d_{damping}\) is applied in the update step; \(d_{damping} = 0.2\) showed good convergence.

IV. OPTIMIZATION RESULTS

The cogging torque optimization is conducted for the presented three-phase outrunner generator. Simulation results obtained by a 2-D FEM simulation of the described iterative optimization algorithm are given in the following. In Fig. 9 (a), the initial geometry is shown and Fig. 9 (b) shows the

Fig. 9: Outrunner generator geometry. (a) initial geometry fed into the optimization and (b) resulting optimized stator geometry with sinusoidal cogging torque.

It is set \(a_1 = 0.5\) in the presented results.

The update of geometry parameters is a multi-dimensional Secant Method, which is related to the multi-dimensional Newton-Raphson Method [24], but utilizes the secant as an approximation for the Jacobian matrix,
resulting geometry obtained by the optimization. Moreover, Fig. 10 shows the resulting air gap function and actual teeth geometry of the optimized design in a linear diagram. It can be observed that the gap between the PMs ($\alpha_{PM_{gap}}$) perceptibly increased, while the stator shape is changed only slightly. A comparison of initial geometry parameters and optimized geometry parameters is given in Table II.

The evolution of cogging torque during the optimization process is depicted in Fig. 11 (for a selected number of designs during the iteration). The optimization converges towards the desired sinusoidal cogging torque, however, the introduced algorithm allows in principle to optimize for any desired cogging torque shape.

A comparison of cogging torques of harvester $T_{cog, harv}$ and the final design of the generator $T_{cog, gen}$ is given in Fig. 12. In summary, the conducted 2-D FEM simulations indicate that the presented algorithm allows to reduce the total cogging torque of the total energy harvester, comprising KEH and outrunner generator, by $\approx 90\%$.

### A. Post-Smoothed Geometry

In order to explain the influence of higher order air gap modulation components ($a_{i>2}$, $b_{i>2}$) further, the optimized geometry of Fig. 9 (b) is smoothed and its cogging torque is analyzed. Moreover, a rather smooth stator geometry could also be a manufacturing requirement, when the sheets of the stator iron are punched in a mass production.

The smoothing is applied to the optimized air gap function $g(\theta)$, depicted in Fig. 10. The resulting cogging torque curve is shown in Fig. 11, where a slight deviation to the desired cogging torque shape.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial Value</th>
<th>Optimized Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{FE}$</td>
<td>60°</td>
<td>59.7°</td>
</tr>
<tr>
<td>$\alpha_{PM_{gap}}$</td>
<td>45°</td>
<td>54.4°</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0</td>
<td>0.0056</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0</td>
<td>0.0245</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0</td>
<td>0.0021</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0</td>
<td>0.0388</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0</td>
<td>0.0073</td>
</tr>
<tr>
<td>$a_4$</td>
<td>0</td>
<td>-0.0356</td>
</tr>
<tr>
<td>$b_4$</td>
<td>0</td>
<td>0.0057</td>
</tr>
<tr>
<td>$a_5$</td>
<td>0</td>
<td>-0.0657</td>
</tr>
<tr>
<td>$b_5$</td>
<td>0</td>
<td>0.0020</td>
</tr>
<tr>
<td>$a_6$</td>
<td>0</td>
<td>-0.0161</td>
</tr>
<tr>
<td>$b_6$</td>
<td>0</td>
<td>0.0030</td>
</tr>
<tr>
<td>$a_{i&gt;6}$, $b_{i&gt;6}$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 12: Compensation of KEH cogging torque ($T_{cog, harv}$) with optimized counter cogging torque of the outrunner generator ($T_{cog, gen}$). The 2-D FEM simulation of generator cogging torque shows that a desired sinusoidal cogging torque pattern is achieved with the optimization. $T_{cog, rem}$ is the resulting total cogging torque of the electromechanical energy harvester, where a reduction of 90%, compared to the cogging torque caused by the KEH ($T_{cog, harv}$) is achieved.
Fig. 13: Map of peak flux density in the stator obtained by 2-D FEM simulation and utilized to compute the iron losses for the efficiency calculation.

sinusoidal form can be noticed. In this case study, smoothing applied on the optimized geometry degraded the cogging torque compensation to $\approx 70\%$.

V. ELECTRICAL PERFORMANCE

The electrical performance of the generator whose cogging torque is optimized in Sec. IV concludes the analysis conducted in this work and will be discussed briefly in the following section.

The generator operation point (OP) is defined by the energy harvesting capability of the KEH. Since various parameters as MCS speed $v_2$, KEH air gap $g_{KEH}$ (cf. Fig. 3 for both) and MCS material grade are taking influence on the generator’s OP, parameters for a nominal OP, based on earlier KEH measurements, are listed in Table III and will serve as a reference OP for characterizing the generator’s performance.

The electrical performance of the generator is calculated regarding only two loss components: iron losses and copper losses. Iron losses $P_{Fe}$ are calculated based on the Steinmetz equation, obtained from the loss data provided by a stator manufacturer for the material M235-35A (cf. EN 10106; with 0.35 mm lamination thickness), as

$$P_{Fe} = \frac{l_{Fe}}{1m} \int_{A_{stator}} 45.7 W/m^2 \cdot \left( \frac{f_{el}}{1 Hz} \right)^{1.30} \cdot \left( \frac{B_{pk}(A)}{1 T} \right)^{1.87} dA \quad (17)$$

where the peak flux density $B_{pk}$ of every point in the stator (shown in Fig. 13) during one electrical period is extracted from 2-D FEM simulations.

The copper losses $P_{cu}$ are calculated as Joule losses due to the required current, where the direct-component of the current is assumed to be zero ($i_d = 0$) for maximum-torque-per-ampere operation. Since the machine’s length is considerably short, end winding losses also contribute significantly ($\approx 50\%$) to the copper losses. Proximity losses in the stator windings can be neglected as the distinct T-shape of the stator teeth effectively shields the winding area.

Finally, the generator’s efficiency for the nominal OP is calculated as $\eta = 90.5\%$, showing that the cogging torque shaping as proposed in this paper does not significantly degrade the performance.

TABLE III: Electrical performance calculation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal torque</td>
<td>$T_n$</td>
<td>45 Nmm</td>
</tr>
<tr>
<td>Nominal speed</td>
<td>$n_m$</td>
<td>2150 rpm</td>
</tr>
<tr>
<td>Nominal mechanical power</td>
<td>$P_{mech}$</td>
<td>10.1 W</td>
</tr>
<tr>
<td>Winding cross-section</td>
<td>$A_{winding}$</td>
<td>143 mm$^2$</td>
</tr>
<tr>
<td>Filling factor</td>
<td>$k_{Cu}$</td>
<td>0.65</td>
</tr>
<tr>
<td>Total copper volume</td>
<td>$V_{Cu}$</td>
<td>5.2 cm$^3$</td>
</tr>
<tr>
<td>Number of turns</td>
<td>$N$</td>
<td>120</td>
</tr>
<tr>
<td>Phase current</td>
<td>$I_{RMS}$</td>
<td>0.69 A</td>
</tr>
<tr>
<td>Average coil flux</td>
<td>$\Phi_{RMS}$</td>
<td>90 $\mu$Wb</td>
</tr>
<tr>
<td>Iron losses</td>
<td>$P_{Fe}$</td>
<td>0.2 W</td>
</tr>
<tr>
<td>Copper losses</td>
<td>$P_{Cu}$</td>
<td>0.7 W</td>
</tr>
<tr>
<td>Efficiency</td>
<td>$\eta$</td>
<td>90.5%</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

This paper describes a method for shaping the cogging torque of a PM outrunner generator towards a desired sinusoidal torque pattern. The underlying goal of this approach is to compensate the cogging torque of a kinetic energy harvester (KEH, [18]) with the optimized counter cogging torque of the generator mechanically connected to the same shaft. Therefore, the total cogging torque can be highly reduced and a self-starting electromechanical energy harvester, comprising the KEH and the outrunner generator can be formed.

The key degree of freedom for shaping the cogging torque is the sinusoidal modulation of the machine’s air gap. An algorithm based on the multi-dimensional Secant Method, which is related to the multi-dimensional Newton-Raphson Method, first evaluates the cogging torque of the current generator geometry with two-dimensional Finite Element Method (2-D FEM) simulations and then iterates the geometry of the outrunner machine until the cogging torque target is achieved. The air gap coefficients $a_i$ and $b_i$ describe the angular modulation of air gap with a sinusoidal basis, similar to a Fourier decomposition. Moreover, stator tooth width $a_{FE}$ and angular distance between the rotor PMs $\alpha_{PMgap}$ are considered as geometry design parameters for the optimization.
The optimization results in a generator design with the desired sinusoidal cogging torque pattern and/or allows to achieve a total cogging torque reduction of the electromechanical energy harvester of 90%. Moreover, a brief electrical performance calculation reveals that the cogging torque shaping as proposed in this paper does not significantly degrade the machine’s efficiency, which is calculated as $\eta = 90.5\%$ for the nominal operating point.

The presented cogging torque optimization is applied for an outrunner generator, providing self-starting capability of an electromechanical energy harvester, however, it can be a helpful tool for emerging applications in areas not directly linked to energy harvesting. A general, undesired repetitive torque ripple, induced by e.g. a mechanical power source, a mechanical load system or by the machine drive system can be eliminated by the counter cogging torque of an accordingly optimized PM machine.

VII. ACKNOWLEDGMENT

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